

Fig. 2. Circumferential stress contours around the notch border (0.3-2-50)

based on a two-dimensional linear elastic Finite Element (FE) analysis which is performed by ABAQUS 6-13 software.

### 3- 1- ASED criterion

According to ASED criterion, failure occurs when the average value of the strain energy density over a specified control volume reaches the critical SED value. Two independent material parameters, namely the critical strain energy density ( $W_{cr}$ ) and the radius of the control volume ( $R_c$ ) are needed for applying this fracture criterion. These parameters are obtained by substituting the material properties available in reference [4] into Eqs. (1) and (2). Therefore, the two independent parameters for GPPS are  $R_c=0.5297$  mm and  $W_{cr}=0.145$  MJ/m<sup>3</sup>.

$$R_c = \frac{(1+\nu)(5-8\nu)}{4\pi} \left(\frac{K_{Ic}}{\sigma_u}\right)^2 \quad (1)$$

$$W_{cr} = \frac{\sigma_u^2}{2E} \quad (2)$$

### 3- 2- ASED-EFC criterion

Equivalent expressions of the averaged strain energy density for mode I and mixed mode I/II loadings are proposed in Eqs. (3) and (4) in which parameters  $W$ ,  $\sigma_{max}$  and the  $H$  constants are the ASED value, maximum tangential stress and equivalent factors, respectively. Also, the subscripts I and I/II denote the mode I and mixed mode I/II loadings, respectively.

$$W_I = H(R_c, \rho) \frac{(\sigma_{max(I)})^2}{2E} \quad (3)$$

$$W_{I/II} = H^*(R_c, \rho) \frac{(\sigma_{max(I/II)})^2}{2E} \quad (4)$$

After a large number of FE calculations, it is found that the parameter  $H$  in Eq. (3) could be utilized instead of the  $H^*$  in Eq. (4) because the difference between these two parameters is not considerable. Based on this result, an approximate procedure is used to evaluate the critical load ( $P_{cr}$ ) for mixed mode I/II loading by means of Eqs. (5) and (6) in which the parameter  $t_k$  is equal to  $\sigma_{max}/(1N)$ . Also, by eliminating the parameter  $H$ , Eq. (6) is derived. As seen in Eq. (6), the parameter  $P_{cr}$  is conveniently calculated for mixed mode I/II loading by applying the values of the averaged strain energy density for mode I loading ( $W_I$ ) and the maximum tangential stress ratio between mode I and mixed mode I/II loadings. Such formulations briefly represent the ASED-EFC criterion. Hence, using the ASED-EFC criterion enables engineers to estimate the experimental fracture loads by means of the reduced numerical calculations.

$$W_{cr} = H(R_c, \rho) \frac{[\sigma_{max}(P_{cr})]^2}{2E} \quad (5)$$

$$P_{cr} = \frac{\sigma_{max(I)}}{t_k} \sqrt{\frac{W_{cr}}{W_I}} \quad (6)$$

## 4- Results and Discussion

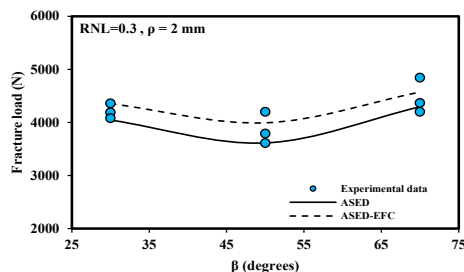
All the experimental and theoretical fracture loads are summarized in Table 1.

Table 1. The experimental and theoretical results of the fracture load for the tested GPPS specimens

d/D- $\rho$ - $\beta$	$P_{av.}, N$ EXP	$P_{Theor.}, N$ ASED	$P_{Theor.}, N$ ASED-EFC
0.3-1-0	4954	5060	5060
0.3-1-30	3946	3780	3995
0.3-1-50	3208	3587	3775
0.3-1-70	4186	4640	5013
0.3-2-0	4964	4947	4947
0.3-2-30	4209	4045	4354
0.3-2-50	3867	3614	3993
0.3-2-70	4470	4295	4572
0.3-4-0	4655	4532	4532
0.3-4-30	4593	4157	4284
0.3-4-50	4364	3783	4036
0.3-4-70	4457	3894	4157
0.5-1-0	3415	3403	3403
0.5-1-30	1917	2286	2451
0.5-1-50	2459	2638	2790
0.5-1-70	3875	4092	4621
0.5-2-0	3152	3279	3279
0.5-2-30	2299	2365	2598
0.5-2-50	2637	2450	2691
0.5-2-70	3374	3263	3853
0.5-4-0	3055	3024	3024
0.5-4-30	2718	2460	2616
0.5-4-50	2629	2360	2570
0.5-4-70	2980	2950	3143
0.5-6-0	2802	2822	2822
0.5-6-30	2847	2449	2546
0.5-6-50	2560	2350	2511
0.5-6-70	2934	2671	2850

The trend of the experimental results as well as the theoretical predictions which obtained from the two energy-based criteria show that the fracture load first decreases and then increases as  $\beta$  enhances from 30° to 70° (see for instance Fig. 3). As seen in Table 1, similar trend is observed for the other tested GPPS specimens with different notch geometries and loading angles. Applying a constant load to the FE model and performing the energy-based FE analysis for each specimen

under various loading conditions shows that the value of the ASED over a specified control volume first increases and then decreases meaning that the fracture load first decreases and then increases. In fact, this description is a justification for fracture behavior of the Key-BD specimens under compressive-shear loading.



**Fig. 3. Comparison of the theoretical predictions and experimental data for the specimen of RNL=0.3,  $\rho=2$  mm**

### 5- Conclusions

Mixed mode I/II brittle fracture were investigated in key-hole notched specimens both experimentally and theoretically.

The experimental fracture loads were predicted by means of the two energy-based criteria, namely the ASED and ASED-EFC criteria. It was found that both criteria could successfully predict the experimental results.

### References

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