



# Wave Propagation in Embedded Temperature-dependent Functionally Graded Nano-plates Subjected to Nonlinear Thermal Loading According to a Nonlocal Four-variable Plate Theory

F. Ebrahimi\*, A. Dabbagh

Department of Mechanical Engineering, Faculty of Engineering, Imam Khomeini International University, Qazvin, Iran.

**ABSTRACT:** In this article, an analytical approach is used to study the effects of thermal loading on the wave propagation characteristics of an embedded functionally graded nano-plate based on refined four-variable plate theory. The heat conduction equation is solved to derive the nonlinear temperature distribution across the thickness. Temperature-dependent material properties of nano-plate are graded using Mori-Tanaka model. The nonlocal elasticity theory of Eringen is introduced to consider small-scale effects. The governing equations are derived by means of Hamilton's principle. Obtained frequencies are validated with those of previously published works. Moreover, effects of different parameters such as temperature distribution, foundation parameters, nonlocal parameter and gradient index on the wave propagation response of size-dependent functionally graded nano-plates have been investigated.

**Review History:**

Received: 30 July 2016  
Revised: 18 January 2017  
Accepted: 5 March 2017  
Available Online: 22 March 2017

**Keywords:**

Nonlocal wave propagation  
Four-variable plate theory  
Inhomogeneous nano-plate  
Mori-Tanaka model  
Thermal effects

**1. Introduction**

Functionally Graded Materials (FGMs) are a new class of composite materials with many enhanced material properties which results in a large use of FGMs in several engineering fields [1]. Nanoscale structures are of an extreme importance in the field of nano-mechanics, so it is crucial to account for small scale effects in their mechanical analysis. Hence, size dependent continuum theories such as nonlocal elasticity theory of Eringen [2] is developed to consider the small scale effects. Employing nonlocal Elasticity, Wang et al. [3] studied wave propagation behaviors of nano-plates. Narendar [4] studied the thermal effects on ultrasonic wave propagation characteristics of nano-plates. Recently, static and dynamic analysis of nonlocal FG nanostructures is of significance [5, 6]. In other words, more exact plate theories are required to estimate the response of plates, therefore, Higher-order Shear Deformation Theories (HSDTs) are suggested and used in some of the researches [7, 8].

Reviewing literature shows that there are only a few articles dealing with wave propagation problem of FG nano-plates. Some of the newly performed attempts have been observed investigating wave dispersion answers of FG nano-beams [9, 10]. In this paper, the nonlocal elasticity is employed to examine the wave propagation behavior of size-dependent FG nano-plates resting on Winkler-Pasternak substrate subjected to thermal loading using a refined higher-order plate theory.

**1- Methodology**

Here, the displacement fields are presumed to be like Barati et al. [8]. Material properties can be calculated utilizing Mori-Tanaka scheme as written in Ref. [8]. Furthermore, a nonlinear

distribution for temperature is defined and the material properties are used with temperature-dependent properties [5]. Hamilton's principle is applied and the Euler-Lagrange equations of a refined third-order plate are computed. The nonlocal elasticity theory is implemented to show influences of small size. Finally, the nonlocal governing equations of FG nano-plates are written as follows:

$$\begin{pmatrix} A_{11} \frac{\partial^2 u_0}{\partial x^2} + (A_{12} + A_{66}) \frac{\partial^2 v_0}{\partial x \partial y} + A_{66} \frac{\partial^2 u_0}{\partial y^2} - B_{11} \frac{\partial^3 w_b}{\partial x^3} \\ -(B_{12} + 2B_{66}) \frac{\partial^3 w_b}{\partial x \partial y^2} - B_{11} \frac{\partial^3 w_s}{\partial x^3} - (B_{12}^s + 2B_{66}^s) \frac{\partial^3 w_s}{\partial x \partial y^2} \end{pmatrix} \quad (1)$$

$$+ (1 - \mu \nabla^2) \left( -I_0 \frac{\partial^2 u_0}{\partial t^2} + I_1 \frac{\partial^3 w_b}{\partial x \partial t^2} + J_1 \frac{\partial^3 w_s}{\partial x \partial t^2} \right) = 0$$

$$\begin{pmatrix} A_{22} \frac{\partial^2 v_0}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 u_0}{\partial x \partial y} + A_{66} \frac{\partial^2 v_0}{\partial x^2} - B_{22} \frac{\partial^3 w_b}{\partial y^3} \\ -(B_{12} + 2B_{66}) \frac{\partial^3 w_b}{\partial x^2 \partial y} - B_{22}^s \frac{\partial^3 w_s}{\partial y^3} - (B_{12}^s + 2B_{66}^s) \frac{\partial^3 w_s}{\partial x^2 \partial y} \end{pmatrix} \quad (2)$$

$$+ (1 - \mu \nabla^2) \left( -I_0 \frac{\partial^2 v_0}{\partial t^2} + I_1 \frac{\partial^3 w_b}{\partial y \partial t^2} + J_1 \frac{\partial^3 w_s}{\partial y \partial t^2} \right) = 0$$

$$\begin{pmatrix} B_{11} \frac{\partial^3 u_0}{\partial x^3} + (B_{12} + 2B_{66}) \frac{\partial^3 u_0}{\partial x \partial y^2} + B_{22} \frac{\partial^3 v_0}{\partial y^3} + (B_{12} + 2B_{66}) \frac{\partial^3 v_0}{\partial x^2 \partial y} \\ -D_{11} \frac{\partial^4 w_b}{\partial x^4} - 2(D_{12} + 2D_{66}) \frac{\partial^4 w_b}{\partial x^2 \partial y^2} - D_{22} \frac{\partial^4 w_b}{\partial y^4} - D_{11}^s \frac{\partial^4 w_s}{\partial x^4} \\ -2(D_{12}^s + 2D_{66}^s) \frac{\partial^4 w_s}{\partial x^2 \partial y^2} - D_{22}^s \frac{\partial^4 w_s}{\partial y^4} \end{pmatrix} \quad (3)$$

$$+ (1 - \mu \nabla^2) \left( -I_0 \frac{\partial^2 (w_b + w_s)}{\partial t^2} - I_1 \left( \frac{\partial^3 u_0}{\partial x \partial t^2} + \frac{\partial^3 v_0}{\partial y \partial t^2} \right) + I_2 \nabla^2 \frac{\partial^2 w_b}{\partial t^2} \right. \\ \left. + J_2 \nabla^2 \frac{\partial^2 w_s}{\partial t^2} + (k_p - N^T) \nabla^2 (w_b + w_s) - k_w (w_b + w_s) \right) = 0$$

Corresponding author, E-mail: r\_moharami@znu.ac.ir

$$\begin{aligned} & \left( B_{11}^s \frac{\partial^3 u_0}{\partial x^3} + (B_{12}^s + 2B_{66}^s) \frac{\partial^3 u_0}{\partial x \partial y^2} + B_{22}^s \frac{\partial^3 v_0}{\partial y^3} + (B_{12}^s + 2B_{66}^s) \frac{\partial^3 v_0}{\partial x^2 \partial y} \right. \\ & \left. - D_{11}^s \frac{\partial^4 w_b}{\partial x^4} - 2(D_{12}^s + 2D_{66}^s) \frac{\partial^4 w_b}{\partial x^2 \partial y^2} - D_{22}^s \frac{\partial^4 w_b}{\partial y^4} - H_{11}^s \frac{\partial^4 w_s}{\partial x^4} \right. \\ & \left. - 2(H_{12}^s + 2H_{66}^s) \frac{\partial^4 w_s}{\partial x^2 \partial y^2} + A_{44}^s \nabla^2 w_s \right) \\ & + (1 - \mu \nabla^2) \left( -I_0 \frac{\partial^2 (w_b + w_s)}{\partial t^2} - J_1 \left( \frac{\partial^3 u_0}{\partial x \partial t^2} + \frac{\partial^3 v_0}{\partial y \partial t^2} \right) + J_2 \nabla^2 \frac{\partial^2 w_b}{\partial t^2} \right. \\ & \left. + K_2 \nabla^2 \frac{\partial^2 w_s}{\partial t^2} + (k_p - N^T) \nabla^2 (w_b + w_s) - k_w (w_b + w_s) \right) = 0 \end{aligned} \quad (4)$$

At last, by solving the above equations analytically the wave frequency of FG nano-plates can be calculated for each mode.

## 2- Results and Discussion

The stiffness-softening produced by nonlocal parameter can be seen here. It is obvious that once  $\Delta T$  increases value of phase velocity and wave frequency decreases. Influences of elastic foundation can be well seen; Winkler and Pasternak parameters both strengthen phase velocities. Furthermore, it is clear that by increasing gradient index the answers diminish continuously. Herein, a series of diagrams are shown for the first mode.

## 3- Conclusions

It can be observed that presented results emphasize that all of the using an elastic medium can make values of phase velocity greater. Moreover, if nonlocal parameter is increased, wave frequency decreases. Also, foundation coefficients can strengthen such variables. Also,  $\Delta T$  has a reverse relation with phase velocity of FG nano-plate.

## References

- [1] Birman, V., Byrd, L. W. (2007). "Modeling and analysis of functionally graded materials and structures." *Appl Mech Rev*, 60, 195-216.
- [2] Eringen, A. Cemal. (1983). "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves." *Journal of applied Physics* 54.9: 4703-4710.
- [3] Wang, Y., Li, F., Kishimoto, K. (2010). "Scale effects on flexural wave propagation in nanoplate embedded in elastic matrix with initial stress." *Appl Phys A*, 99, 907-911.
- [4] Narendar, S., Gopalakrishnan, S. (2011). "Temperature effects on wave propagation in nanoplates." *Composites: Part B*, 43, 1275-1281.
- [5] Ebrahimi, F., Salari, E. (2015). "Thermal buckling and free vibration analysis of size dependent Timoshenko FG nanobeams in thermal environments." *Composite Structures*, 128, 363-380.
- [6] Ebrahimi, F., Salari, E., Hosseini, S. A. H. (2015). "Thermomechanical vibration behavior of FG nanobeams subjected to linear and nonlinear temperature distributions." *Journal of Thermal Stresses*, 38(12), 1360-1386.
- [7] Tounsi, A., Ait Yahia, S., Ait Atmane, H., Houari, M. S. A. (2014). "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories." *Structural Engineering and Mechanics*, 53, 1143-1165.
- [8] Barati, M. R., Zenkour, A. M., Shahverdi, H. (2016). "Thermo-mechanical buckling analysis of embedded nanosize FG plates in thermal environments via an

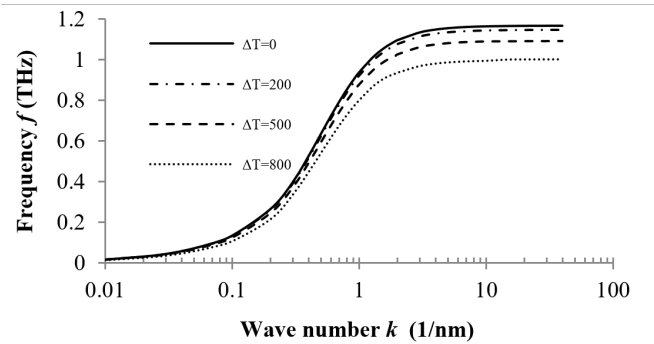


Fig. 1. Effect of various temperature changes on the wave frequency of FG nanoplates ( $p=1, \mu=1 \text{ nm}^2$ ).

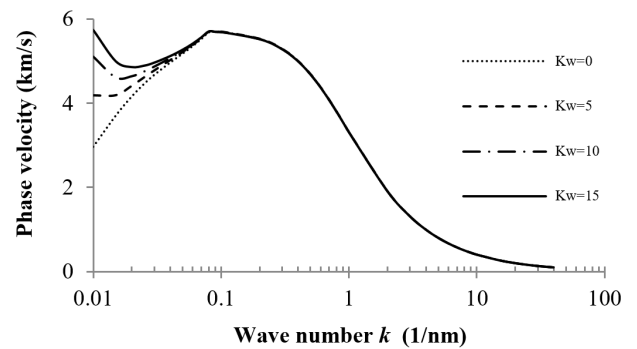


Fig. 2. Effect of Winkler parameter on the phase velocity of FG nanoplates ( $p=1, \mu=1 \text{ nm}^2, \Delta T=500$ ).

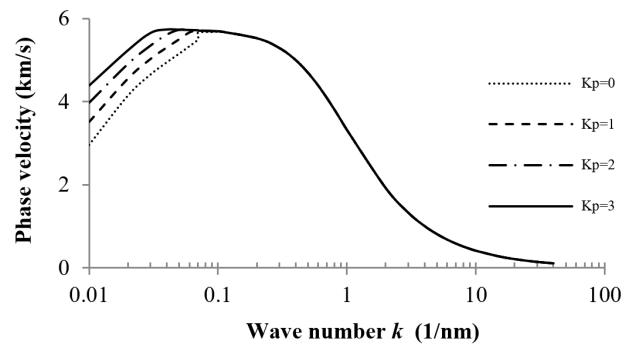


Fig. 3. Effect of Pasternak parameter on the phase velocity of FG nanoplates ( $p=1, \mu=1 \text{ nm}^2, \Delta T=500$ ).

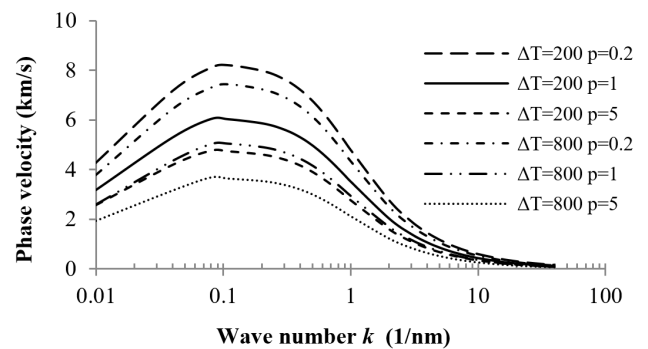


Fig. 4. Effects of gradient index and temperature change on the phase velocity of the FG nanoplates.

inverse cotangential theory.” *Composite structures*, 141, 203-212.

[9] Zhang, Y., Chen, J., Reddy, J. N. (2015). “Surface and thermal effects on the flexural wave propagation of piezoelectric functionally graded nanobeam using nonlocal elasticity.” *Computational Materials Science*,

97, 22-226.

[10] Li, Li., Hu, Y., Ling, L. (2015). “Flexural wave propagation in small-scaled functionally graded beams via a nonlocal strain gradient theory.” *Composite Structures*, 133, 1079-1092.

Please cite this article using:

F. Ebrahimi, A. Dabbagh, Wave Propagation in Embedded Temperature-dependent Functionally Graded Nano-plates Subjected to Nonlinear Thermal Loading According to a Nonlocal Four-variable Plate Theory, *Amirkabir J. Mech. Eng.*, 50(5) (2018) 317-320.

DOI: 10.22060/mej.2017.11824.5194



