



## Investigation of Viscoelastic-Viscoplastic-Viscodamage Behavior of Polymers under Cyclic Loadings and Parametric Study of Their Mechanical Behavior

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**ABSTRACT:** Nowadays, polymers have attracted many researchers due to individual properties in responding to mechanical loadings. According to their time and rate-dependent response to the exterior loads, there was too much complexity to predict mechanical responses. Thus, a thermodynamically consistent viscoelastic-viscoplastic-viscodamage model is considered to predict their mechanical responses. Along with this purpose, the explicit time-discrete form of the constitutive model is used in the finite element software ABAQUS by VUMAT subroutine. This method is valuable since its accuracy, low running time and no need for Jacobian matrix calculations. After validating ABAQUS user subroutine with experimental data obtained from creep, creep-recovery and repeated creep-recovery tests on the polymer, constitutive model sensitivity to the material parameters such as viscoelastic, viscoplastic and damage properties has been investigated. Along with this purpose, it can be observed that increasing viscoplastic and damage parameters could cause rising in the viscoplastic strain and damage variable. Then, the direct effects of loading time and stress level and the indirect effects of unloading time on the polymer strain and damage variable were investigated in the two repeated creep-recovery tests with constant and increasing amplitudes. For instance, doubling the increasing amplitude cycle number from 50 to 100 could rise service life and decrease about 75 percent of damage level.

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### 1. Introduction

Nowadays, polymers according to their properties such as high strength to weight ratio, resilient, transparency, toughness, color and processing, causes the industries to grow faster. Time and rate dependency are individual characteristics which make polymers differ from other materials. Therefore, their mechanical properties vary with time [1]. Many experimental studies indicated that the time-dependent behavior of polymers could be divided into reversible and irreversible parts. These reversible and irreversible deformations could be simulated by viscoelastic and viscoplastic models [2].

According to the continuum damage mechanics theories, when the loading applies on the material sample, damage appears in the material structure which affects its properties. Thus, the mechanical properties of polymers are the function of the damage variable [2, 3].

Many of the constitutive models are based on the continuum damage mechanics. Investigating mechanical behaviors is performed by considering viscoelastic and viscoplastic properties for these models. Al-Rub et al.[3] predicted viscoelastic-viscoplastic-viscodamage behavior of polymers under the fatigue loading by presenting the continuum damage mechanic constitutive model. Next, they validated this constitutive model by investigating the uniaxial repeated creep-recovery test data.

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In this paper, we have presented the explicit time discretization of the consistent viscoelastic-viscoplastic-viscodamage thermodynamic model [4]. The time-dependent mechanical behavior of polymers is predicted by this constitutive model. The discretization has been used in finite element subroutine which has been validated by creep, creep recovery and repeated creep recovery experimental data. Due to the importance of every division of this constitutive model, sensitivity parametric study and their effects on mechanical behavior of polymers has been investigated. Eventually, the effects of the applied loading characteristics such as loading time, unloading time and stress level on the mechanical behavior of these materials were studied under two repeated creep recovery loading with constant and increasing amplitude.

### 2. Constitutive Equations and Time Discretization

Due to the small strain assumption of this constitutive model [4], total strain decomposed to the elastic ( $e$ ), viscoelastic ( $ve$ ) and viscoplastic ( $vp$ ) parts.

$$\tilde{\boldsymbol{\epsilon}} = \tilde{\boldsymbol{\epsilon}}^e + \tilde{\boldsymbol{\epsilon}}^{ve} + \tilde{\boldsymbol{\epsilon}}^{vp} \quad (1)$$

Next, deviatoric ( $e$ ) and volumetric ( $\theta$ ) parts of viscoelastic and viscoplastic strain evolution are obtained by satisfying the Clausius-Duhem inequality:



$$\begin{aligned} \dot{\mathbf{e}}^{vp} &= \dot{\lambda} \frac{\tilde{\mathbf{s}}_2}{\|\tilde{\mathbf{s}}_2\|}, & \dot{\theta}^{vp} &= -\dot{\lambda}\beta, \\ \dot{\mathbf{e}}^{ve} &= \frac{1}{\zeta_e} \tilde{\mathbf{s}}_1, & \dot{\theta}^{ve} &= \frac{1}{\zeta_\theta} \tilde{p}_1. \end{aligned} \quad (2)$$

where  $\zeta_e$  is the deviatoric and  $\zeta_\theta$  volumetric viscosity parameters,  $\dot{\lambda}$  is a Lagrangian multiplier,  $\beta$  is a material parameter and  $\tilde{p}_1$ ,  $\tilde{\mathbf{s}}_1$  and  $\tilde{\mathbf{s}}_2$  are the relative stresses. In addition, damage evolution is considered as:

$$\dot{D} = \Gamma^{vd} \left( \frac{\tilde{Y}_{eq}}{Y_0} \right)^{X_1} (1-D)^{X_2} \exp(K_D \tilde{\epsilon}_{eq}) \quad (3)$$

where  $X_1$ ,  $X_2$  and  $K_D$  are the material parameters.  $\Gamma^{vd}$ ,  $\tilde{Y}_{eq}$  and  $Y_0$  indicate the evolution amplitude and the effective strain is  $\tilde{\epsilon}_{eq} = \sqrt{\tilde{\epsilon}_{ij} \tilde{\epsilon}_{ij}}$ .

The explicit time discretization of the above relations is presented as follows:

$$\begin{aligned} \tilde{\mathbf{e}}_t^{vp} &= \tilde{\mathbf{e}}_{t-\Delta t}^{vp} + \Delta\lambda \left( \frac{\tilde{\mathbf{s}}_{2,t}}{\|\tilde{\mathbf{s}}_{2,t}\|} \right) & \tilde{\mathbf{e}}_t^{ve} &= \tilde{\mathbf{e}}_{t-\Delta t}^{ve} + \frac{\Delta t}{\zeta_e} \tilde{\mathbf{s}}_{1,t} \\ \tilde{\theta}_t^{vp} &= \tilde{\theta}_{t-\Delta t}^{vp} - \beta\Delta\lambda & \tilde{\theta}_t^{ve} &= \tilde{\theta}_{t-\Delta t}^{ve} + \frac{\Delta t}{\zeta_\theta} \tilde{p}_{1,t} \end{aligned} \quad (4)$$

$$D_t = D_{t-\Delta t} + \Gamma^{vd} \left( \frac{\tilde{Y}_{eq}}{Y_0} \right)^{X_1} (1-D_{t-\Delta t})^{X_2} \exp(K_D \tilde{\epsilon}_{eq,t}) \Delta t \quad (5)$$

where the past and the present time step is denoted by subscript  $t - \Delta t$  and  $t$ , respectively.

### 3. Validation

In order to use VUMAT subroutine in finite element software ABAQUS, time discretization results and experimental data of Darabi and his colleagues study [2] were compared under the creep, creep recovery and repeated creep-recovery test.

### 4. Parametric Study, Results, and Discussion

The parametric study of the constitutive models was performed to show the sensitivity of polymers behavior through a variety of mechanical properties. Most of the parameters were investigated under the different loadings (e.g. viscoelastic, viscoplastic and viscodamage variables, stress level and loading time). Here, Fig. 1 indicates the constitutive model sensitivity to viscoplastic viscosity parameter ( $\Gamma^{vd}$ ) under the repeated creep recovery loading.

Due to the main effects of the stress level and loading time on polymers service life, their behavior was indicated under two repeated creep recovery with constant and increasing amplitude as Figs. 2 and 3.

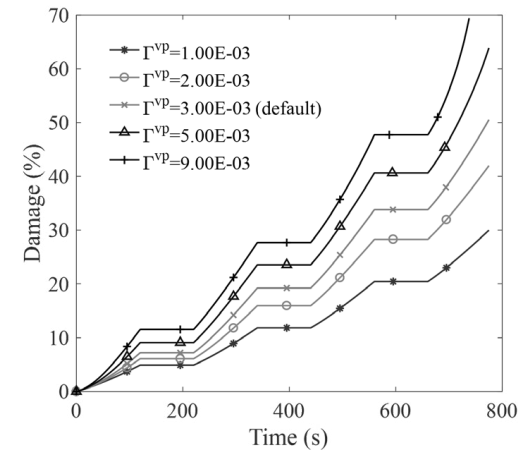
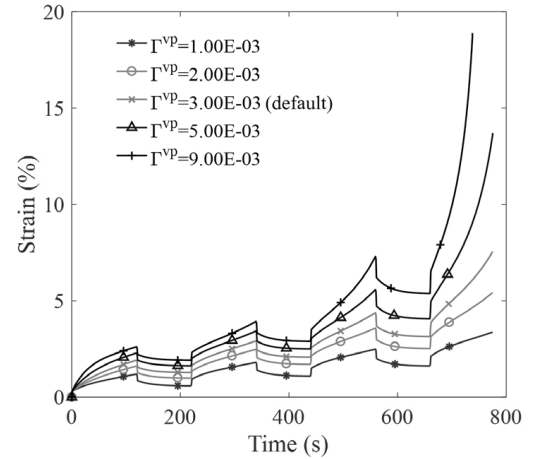


Fig. 1: Model sensitivity to viscoplastic viscosity parameter under repeated recovery a) strain b) damage

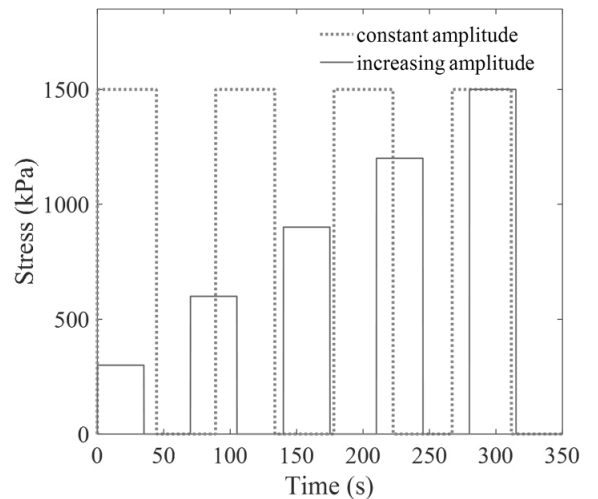
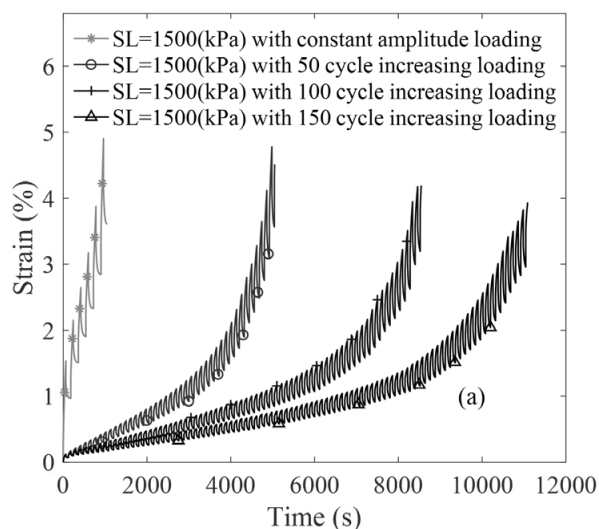


Fig. 2: Schematic comparison of the repeated creep recovery loading with constant and increasing amplitude

By a closer look at the results, we can understand the direct relationship between  $\Gamma^{vd}$  (Fig. 1) and stress level (Fig. 3) with the plastic deformation and damage. Rising these parameters cause the material stiffness and the total strain to decrease and increase, respectively.



**Fig. 3: Schematic comparison of the repeated creep recovery loading with constant and increasing amplitude**

### 5. Conclusion

In this paper, the viscoelastic-viscoplastic-viscodamage constitutive model was discretized and implemented in VUMAT subroutine to be used in FE software. The explicit time discretization was valuable due to its accuracy, low running time and no need for Jacobian matrix calculations.

The sensitivity of polymers behavior was investigated to the viscoelastic, viscoplastic and viscodamage parameters. In addition, the direct effects of the loading characteristics such as loading time and stress level to damage variable were observed, although the unloading time has indirect effects on damage variable.

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