



## Multiscale simulation of flow in fractured porous media using unstructured grids

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**ABSTRACT:** In this paper for flow simulation in fractured porous media, a multiscale finite volume method on unstructured grids is developed. To this end, algorithms for generating coarse scale unstructured grids for the matrix and fracture networks are presented independently. The presented algorithms for generating coarse scale unstructured grids are adaptable based on local changes in permeability field. Unstructured grid adaption based on permeability field has significant effect on improving the multiscale solution results in highly heterogeneous permeability fields. For the first time in this research, applying adaptive unstructured grids in fractured porous media is done. Coarse scale grid cells are generated such that strong variation of permeability along their boundaries and also the placement of coarse scale vertices in low permeability region are prevented. To reduce the computational cost, fracture-matrix coupling is considered only for the calculation of basis functions in the matrix domain. In order to evaluate the proposed algorithms, various 2D test cases are designed and solved. Finally, it is shown that the multiscale finite volume method with the proposed algorithms is an efficient numerical method for flow simulation in heterogeneous fractured porous media.

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## 1. INTRODUCTION

Flow simulation in fractured reservoirs is an important and challenging issue in the oil and gas industry. Various numerical methods have been proposed to investigate the behavior of fractured reservoirs. One of the numerical methods that has been considered is discrete fracture modeling, where fractures are modeled as elements with one dimension less than the matrix [1]. In discrete fracture model using unstructured grids, fractures are placed at the interface between matrix cells [2,3]. In this method due to using unstructured grids, complex geological features including fracture networks are better described. However, for real field application that leads to large linear systems which cannot be solved with the existing classical method. To resolve this problem, multiscale methods have been developed [4].

Although promising progress has been made in recent years, the challenge of generating flexible multiscale unstructured grids for discrete fracture modeling has not yet been addressed. In this paper, the multiscale finite volume method for discrete fracture modeling on unstructured grids is developed. To this end, algorithms for generating adaptive unstructured coarse grids are presented. Generation of unstructured coarse grids is based on local changes in permeability field. Numerical results show that the multiscale finite volume method using adaptive unstructured coarse grids predicts fine-scale solution with high accuracy without using iterative methods.

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## 2. METHODOLOGY

For incompressible flow in porous media with discrete fractures, the pressure equation is described using Darcy's law as:

$$-\nabla \cdot (\lambda \cdot \nabla p) = q \quad (1)$$

where  $\lambda$  is the mobility,  $p$  is pressure and  $q$  represents injection and production wells. Eq. (1) is solved to obtain the pressure field in fractured porous media using discrete fracture modeling where in this model, fractures are considered one dimension less than the matrix (as a line in a two-dimensional matrix and as a surface in a three-dimensional matrix).

The discretized form of Eq. (1) for matrix and fractures can be written as:

$$\begin{bmatrix} A_{mm} & A_{mf} \\ A_{fm} & A_{ff} \end{bmatrix} \begin{bmatrix} p_m \\ p_f \end{bmatrix} = \begin{bmatrix} q_m \\ q_f \end{bmatrix} \quad (2)$$

where subscript  $m$  and  $f$  are intended for the matrix and the fracture, respectively.

Solving the system of Eq. (2) for real-field simulations will have a high computational cost. To resolve this problem,



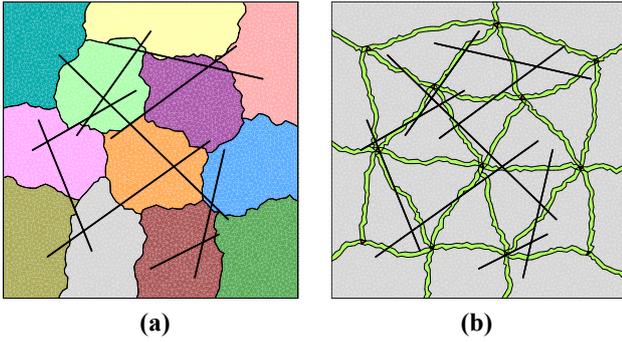


Fig. 1. (a) Primal coarse grid, (b) dual coarse grid in matrix domain

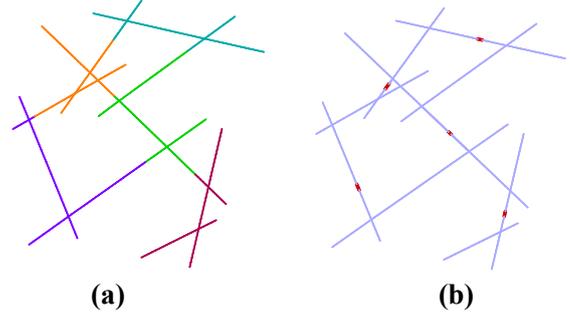


Fig. 2. (a) Primal coarse grid, (b) dual coarse grid in fracture networks

multiscale methods have been introduced. In this paper, the multiscale finite volume method is used to solve the above linear system by providing efficient algorithms.

The multiscale finite volume method uses a Prolongation operator  $\mathcal{P}$  to solve the system of Eq. (2). If  $p^c$  and  $p^{ms}$  are pressure fields on coarse-scale and fine-scale, respectively:

$$p^{ms} = \mathcal{P}p^c \quad (3)$$

To solve the coarse-scale system, a restriction operator  $\mathcal{R}$  is defined that describes the mapping from fine to coarse space:

$$\underbrace{(\mathcal{R}\mathcal{A}\mathcal{P})}_{A^c} p^c = \underbrace{\mathcal{R}q}_{q^c} \quad (4)$$

Finally, the multiscale pressure solution is calculated as:

$$p_f \approx p^{ms} = \underbrace{\mathcal{P}(\mathcal{R}\mathcal{A}\mathcal{P})^{-1}\mathcal{R}q}_{M_{ms}^{-1}} \quad (5)$$

The multiscale finite volume method uses two types of primal and dual coarse grids to calculate the restriction and prolongation operators. In this paper, a multilevel tabu search algorithm is used to generate primal coarse grid. The primal coarse grids for the matrix and fracture domains are generated independently.

At each iteration of the algorithm, the subset with the maximum weight is selected. Then, the boundary vertex with the maximum gain is chosen for migration to the preferred subset. The gain  $g(v,n)$  of vertex  $v$  for migration to the neighboring subset  $S_n$  is defined as

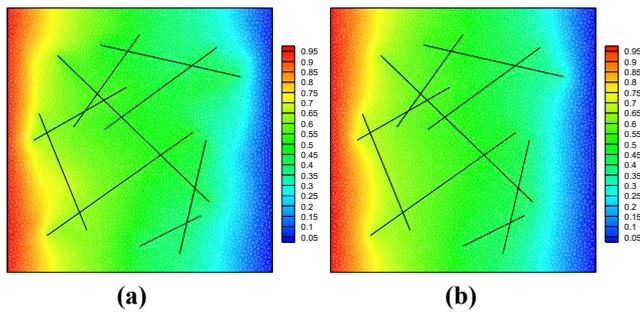
$$g(v,n) = ED[v]_n - ID[v] \quad (6)$$

where  $ED[v]_n$  is the sum of common edge weights between vertex  $v$  and its neighboring vertices located in the subset  $S_n$ , and  $ID[v]$  is the sum of common edge weights between vertex  $v$  and its neighboring vertices in the same subset. After moving each vertex, the gain of the vertex and its neighbors are updated. The algorithm continues until stopping criteria are met.

The proposed algorithm for generating dual coarse grid employs the equivalent graph of unstructured grids. The first step is to specify the vertex cells within each primal coarse cell. Since the presence of vertex cells in a low-permeability region causes nonphysical peaks, the fine cell whose centroid is closest to the mean centroid is selected and its permeability value is checked. If the selected cell is located in a low permeability region, it will be removed from the list of candidate cells. Then, the algorithm searches for the next fine cell and this process continues until the closest cell to the mean centroid with acceptable permeability value is identified. The next step is to assign the edge cells. Dijkstra's algorithm is employed to find the shortest path between the two vertex cells of coarse blocks, so that the strong permeability contrasts along the path of edge cells are minimized. Finally, the interior cells are assigned as the cells that do not belong to the edge and vertex categories.

### 3. NUMERICAL SIMULATION

In this section, the multiscale flow simulation results in fractured porous media using the proposed algorithms for generating multiscale unstructured grids are presented and compared with the fine-scale reference solution. The test case consists of a  $1[m] \times 1[m]$  homogeneous matrix with 9 fractures and  $k_f = 1000k_m$ . Dirichlet boundary conditions are applied on the left and right boundaries with normalized pressures of 1 and 0, respectively and no-flow boundary conditions are specified on the top and bottom boundaries. The fine-scale grid has 10226 matrix and 306 fracture cells and the coarse scale grid contains 12 matrix and 5 fracture cells. Figs. 1(a) and 1(b) present the primal and dual coarse grids for matrix domain. The primal and dual coarse grids for fracture networks are illustrated in Figs. 2(a) and 2(b) respectively.



**Fig. 3. Pressure contours obtained by the (a) MSFV method, (b) fine reference solution**

Figs. 3(a) and 3(b) present the multiscale and fine-scale pressure results. The multiscale finite volume predicts the pressure distribution with relative error  $e_p = 6.48 \times 10^{-2}$ . It is clear that the MSFV solution agrees well with the fine-scale reference solutions.

#### 4. CONCLUSIONS

In this paper, the multiscale finite volume method for discrete fracture modeling on unstructured grids is

developed. Algorithms for generating adaptive unstructured coarse grids are presented. Unstructured coarse-scale grid adaptation based on local changes in permeability field can significantly improve the multiscale solution results in highly heterogeneous permeability fields. Numerical results show that the multiscale finite volume method with the proposed algorithms is an efficient numerical method for flow simulation in heterogeneous fractured porous media.

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