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Superharmonic and Subharmonic Resonance Analysis of A Rectangular Hyperelastic Membrane Resting on Nonlinear Elastic Foundation Using The Method of Multiple Scales

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ABSTRACT: In this paper, the nonlinear vibrations of a rectangular hyperelastic membrane resting on a nonlinear elastic Winkler-Pasternak foundation subjected to uniformly distributed hydrostatic pressure are investigated. The membrane is composed of an incompressible, homogeneous, and isotropic material. The elastic foundation includes two Winkler and Pasternak linear terms and a Winkler term with cubic nonlinearity. Using the theory of thin hyperelastic membrane, Hamilton's principle, and assuming the finite deformations, the governing equations are obtained. Also, the kinetic energy, the work of uniform distributed force and pressure, and the effects of damping are determined, according to the strain energy function for neo-Hookean hyperelastic constitutive law. By applying Galerkin's method, the nonlinear partial differential equation of motion in the transversal direction is transformed to the ordinary differential equations. Then, utilizing the method of multiple scales, the superharmonic and subharmonic resonances including the 1:3 superharmonic and 3:1 subharmonic, 1:5 superharmonic, and 5:1 subharmonic, 1:7 superharmonic, and 7:1 subharmonic are analyzed. Also, the analytical results are compared with those presented by other researchers. Finally, the effect of the Winkler and Pasternak stiffness, the material properties, and various geometrical characteristics on the superharmonic and subharmonic resonances of the vibration behavior of a rectangular hyperelastic membrane is investigated.

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1. INTRODUCTION

The hyperelastic membranes find applications ranging from space structures, actuators, sensors, robotics, and large civil engineering structures to many biological problems and surgical procedures.

Some researches have been focused on the vibration behavior of the hyperelastic structures. For example, the free and forced non-linear vibration of a thin plate with hyperelastic materials was studied by Breslavsky et al. [1]. The vibration behaviors of a radially stretched hyperelastic membrane under finite deformations were investigated by Goncalves et al. [2]. Soares and Goncalves [3] analyzed the nonlinear vibrations of a hyperelastic annular membrane under finite deformations that the material characteristic was similar to research in Ref. [2]. Also, Soares and Goncalves [4] investigated the nonlinear vibration of a rectangular hyperelastic membrane embedded within a nonlinear Winkler-type foundation.

In the most previous works as mentioned above, there has been less attention to the secondary resonance analysis including the superharmonic and subharmonic resonances for the transverse nonlinear vibration of the hyperelastic membranes. Therefore, the main novelties of this study are as follows: (1) Superharmonic and subharmonic resonance analysis of a rectangular hyperelastic membrane is investigated using the method of multiple scales, (2) The rectangular hyperelastic membrane resting on the nonlinear elastic foundation includes two Winkler and Pasternak linear terms and a Winkler term with cubic nonlinearity. The principle of thin hyperelastic membranes is utilized to obtain the differential motion equations.

2. RECTANGULAR HYPERELASTIC MEMBRANE

2.1. Model's geometry

Considering Fig. 1, the schematic of a rectangular hyperelastic membrane is illustrated. The rectangular membrane with density Γ, thickness h, and lengths L_{xo} and L_{yo} is considered. Also, $h/L_{yo}, h/L_{xo} \ll 1$.

According to Fig. 1, a membrane particle is assumed as P_i in the Cartesian coordinate x, y, z, which is transformed to point P_i' due to stretching and then to point P_i'' due to deformation in a Cartesian coordinate $X(x, y, t)$, $Y(x, y, t)$ and $Z(x, y, t)$. The strain energy density function for an isotropic, homogeneous, and rectangular hyperelastic membrane, considering the constitutive law of neo-Hookean is defined as:

$$W = C_1 (I_1 - 3) \tag{1}$$

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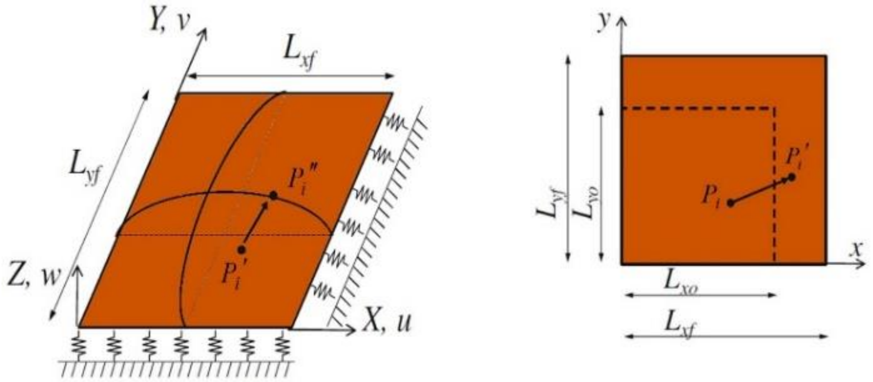


Fig. 1. Schematic of the hyperelastic membrane

where C_1 is the material parameter and I_1 is the first strain invariant that according to the deformation tensor of right Cauchy-Green can be obtained.

3. GOVERNING EQUATIONS

Regarding the undeformed membrane, the total displacements are as follows:

$$\begin{aligned} X &= X_0(x,y)+u(x,y,t) \\ Y &= Y_0(x,y)+v(x,y,t) \\ Z &= w(x,y,t) \end{aligned} \tag{2}$$

where $X_0=\delta_x x$, $Y_0=\delta_y y$, and δ_x δ_y are respectively the stretching ratios in x and y directions.

During the transverse vibration, similar to works presented by other researchers which are according to the finite element method, u and v components can be negligible in comparison with the transverse vibration displacement w [2, 3]. The nonlinear equation of motion in the transverse direction utilizing Hamilton's principle is obtained as follows:

$$\begin{aligned} &-\frac{\partial}{\partial x}\left(h\frac{\partial W}{\partial Z_{,x}}\right)-\frac{\partial}{\partial y}\left(h\frac{\partial W}{\partial Z_{,y}}\right)+h\Gamma\frac{\partial^2 w}{\partial t^2}-C\frac{\partial w}{\partial t}-P_h(t)\delta_x\delta_y \\ &\quad+\left(k_1 w-k_2\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+k_3 w^3\right)\delta_x\delta_y=0 \end{aligned} \tag{3}$$

where $C=\xi C_c$, and ξ C_c are respectively the damping factor and critical damping. $P_h(t)=P_o\cos(\Omega t)$ is the distributed hydrostatic pressure. P_o and Ω are respectively the excitation amplitude and frequency.

3.1. The equation of motion discretization

In this section, to discretize the equation of motion, the transverse vibration displacement is considered as:

$$w(x,y,t)=W(t)\sin\left(\frac{m\pi x}{L_{xo}}\right)\sin\left(\frac{n\pi y}{L_{yo}}\right) \tag{4}$$

where $W(t)$ is the time-dependent modal amplitudes. Substituting Eq. (4) in Eq. (3), and then utilizing Galerkin's method, the discretized nonlinear motion equation in the transverse direction is obtained as:

$$\begin{aligned} &\ddot{W}(t)+\hat{\mu}\dot{W}(t)+\omega_0^2 W(t)+\hat{\alpha}_2 W^3(t) \\ &\quad+\hat{\alpha}_3 W^5(t)+\hat{\alpha}_4 W^7(t)=\eta P_h(t) \end{aligned} \tag{5}$$

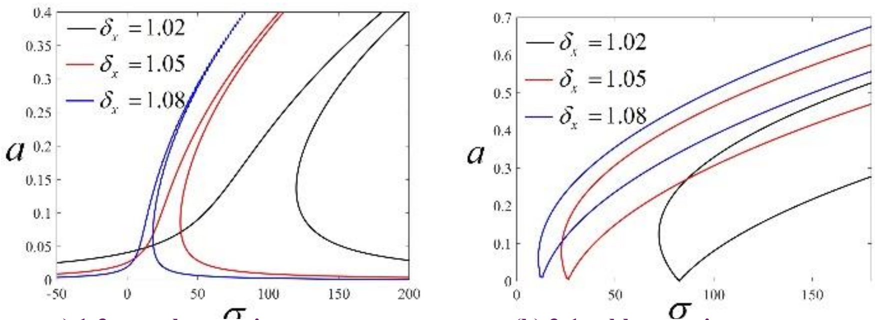
all coefficients are a function of π, h, C_1, Γ, δ_x, δ_y, L_{xo}, L_{yo}, k_1, k_2, and k_3. Although they are too long to be explicitly written here, they can be easily computed with computer algebra.

4. PERTURBATION ANALYSIS

In this section, the secondary resonance cases for the rectangular hyperelastic membrane are analyzed by utilizing the method of multiple scales. In this regard, after finding the solvability equation, the polar form substitutes, then the imaginary and real parts of the resultant equations are obtained. Finally, by calculating the squares of these results and summing them for the steady-state motion, the frequency-response equation can be obtained.

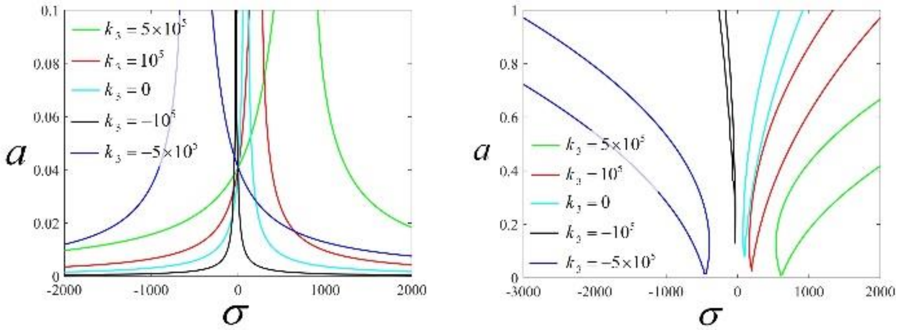
Table 1. The geometrical characteristics and material parameters of the membrane.

Geometrical properties	Value	Material parameters	Value
L_{xo}	1.5 m	Γ	1200 kg/m^3
L_{yo}	1 m	C_1	0.17 MPa
h	0.001 m	-	-



a) 1:3 superharmonic resonance (b) 3:1 subharmonic resonance

Fig. 2. Effect of the stretching ratio on the frequency-response curves



(a) 1:3 superharmonic resonancecurves (b) 3:1 subharmonic resonancecurves

Fig. 3. Effect of the nonlinear stiffness parameter on the frequency-response curves

5. RESULTS AND DISCUSSION

Here, in order to analyze secondary resonances, the material parameters and geometrical characteristics for the neo-Hookean constitutive model are listed in Table 1.

The effect of the various initial stretching ratios in the x-direction (δ_x) on the response of frequency amplitude for 1:3 superharmonic and 3:1 subharmonic resonances are demonstrated in Fig. 2. As can be seen, by increasing δ_x, the curves shift to the left and the hardening nonlinearity behavior is decreased.

The influence of the nonlinear stiffness parameter for Winkler type foundation (k_3) on the frequency response for 1:3 superharmonic and 3:1 subharmonic resonances are demonstrated in Fig. 3. As expressed previously, the coefficient of the nonlinear deflection with cubic nonlinearity (w^3) is considered as softening/hardening cubic nonlinearity parameter for the nonlinear elastic foundation. As shown in the figure, by increasing the value of $k_3 > 0$ and $k_3 < 0$, the hardening nonlinearity behavior is respectively increased and decreased.

6. CONCLUSIONS

In this paper, the secondary resonances for rectangular hyperelastic membrane resting on nonlinear Winkler-Pasternak elastic foundation under harmonic excitation were presented. Considering the theory of thin hyperelastic membrane, Hamilton's principle, and assuming the finite deformations, the problem formulation was obtained. Then, the motion equation in the transverse direction was discretized by applying Galerkin's method. To solve the secondary resonances for different cases, the multiple scales method was utilized. The key results can be summed up as follows:

Increasing stretching ratios in the x and y directions leads to decreasing the hardening nonlinearity behaviors.

By increasing the value of $k_3 > 0$, the hardening nonlinearity behavior is increased, whereas, by increasing the value of $k_3 < 0$, the hardening nonlinearity behavior is decreased.

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