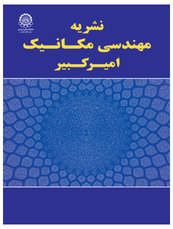

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Investigation Of Nonlinear Vibrations Of A Cantilever Beam With A Transverse Fatigue Crack

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ABSTRACT: Nonlinear vibrations due to the presence of fatigue cracks are suitable indicators for detecting cracks in the structure. Late detection of such cracks may lead to catastrophic failures. Therefore, identifying the behavior of the cracked structure is very important for the prevention of structural failures. In this study, the nonlinear vibration of a cantilever beam with a transverse breathing crack and bilinear behavior has been studied. For this purpose, the restoring force of the cracked beam is considered a nonlinear polynomial function. Then, using the method of multiple scales, the approximated equation of the cracked beam is solved, and the frequency-response curves for both harmonic and superharmonic resonances are extracted. Then, the sensitivity of the responses to the crack depth, crack location, excitation force amplitude, and damping coefficient are investigated. The cracked beam frequency-response curves in the primary resonance have become highly nonlinear due to the increase of the crack parameters and cause softening of the curves. Also, it was observed that the behavior of the beam in superharmonic resonance is highly sensitive to the presence of a fatigue crack in the structure.

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1. INTRODUCTION

Assessment and detection of structural damage such as fatigue cracks, especially in the early stages of its formation, is a vital process in engineering applications to prevent catastrophic failures. Fatigue cracks usually occur in structures exposed to cyclic loads and can significantly reduce the structure's ability to withstand working loads. Many studies have been conducted on the dynamic behavior of fatigue cracks to develop a non-destructive crack identification method and health monitoring of the structures [1-4]. Modeling the dynamic behavior of a cracked structure can be the first step in vibration-based structural health monitoring. For example, identifying the location and depth of cracks in structures such as beams is an important example in engineering applications.

The stiffness of a cantilever beam with a transverse breathing crack changes continuously during the time of oscillation. That is why this type of crack is known as a breathing crack [5, 6]. The presence of a breathing crack in the beam causes a highly nonlinear vibrational response with asymmetric stiffness properties [7]. Under such circumstances, the beam vibration behavior becomes bilinear [8]. Fig. 1 shows a cantilever beam with a transverse breathing crack.

2. METHODOLOGY

2.1. Bilinear behavior of the breathing crack

A beam with a breathing crack acts as a bilinear oscillator during its vibrational cycles, and its response to the harmonic excitation force is a nonlinear response. The equation of motion of the beam with breathing crack (bilinear oscillator) can be written as follows [7]:

$$m\ddot{u}(t)+c\dot{u}(t)+H[u(t)]=f\cos(\Omega t) \tag{1}$$

where $H[u(t)]$ is a bilinear crack function defined as follows [7]:

$$H[u(t)] = \begin{cases} \alpha k_c u & u \geq 0 \\ k_c u & u < 0 \end{cases} \tag{2}$$

2.2. Modeling of a polynomial function of beam stiffness with a breathing crack

The restoring force $H[u(t)]$ can be approximated with a polynomial function of the beam displacement $h[u(t)]$, which can be obtained as follows:

$$h[u(t)] = C_0 + C_1 k_c u + C_2 k_c u^2 + C_3 k_c u^3 \tag{3}$$

Each of the coefficients C_1, C_2 and C_3 are obtained according to equation (4):

$$C_1 = \frac{1}{2}\alpha + \frac{1}{2},\quad C_2 = \frac{5}{8}\frac{\alpha - 1}{X} \quad \text{and} \quad C_3 = 0 \tag{4}$$

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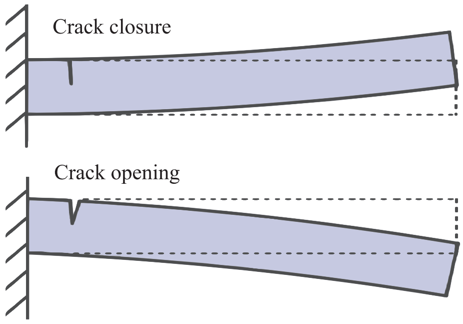

Fig. 1. Cracks opening and closing during the beam oscillation cycle

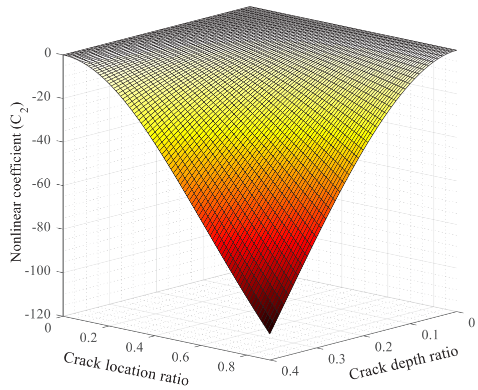

Fig. 2. Changes in the nonlinear coefficient (C_2) for different crack depth and location ratios

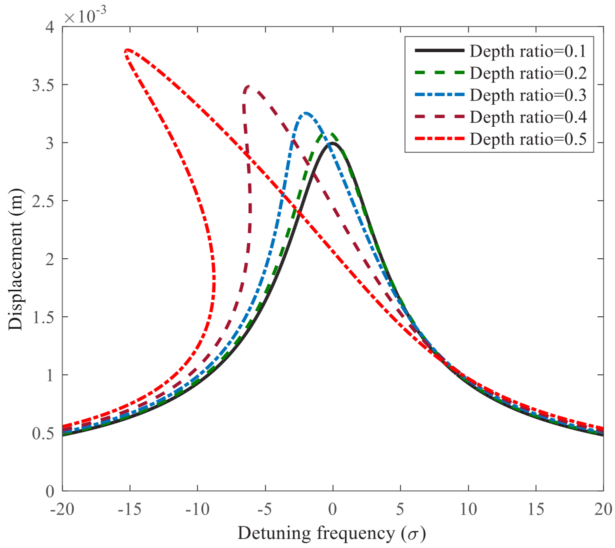

Fig. 3. Frequency-response curves in the primary resonance when location ratio $\eta = 0.9$

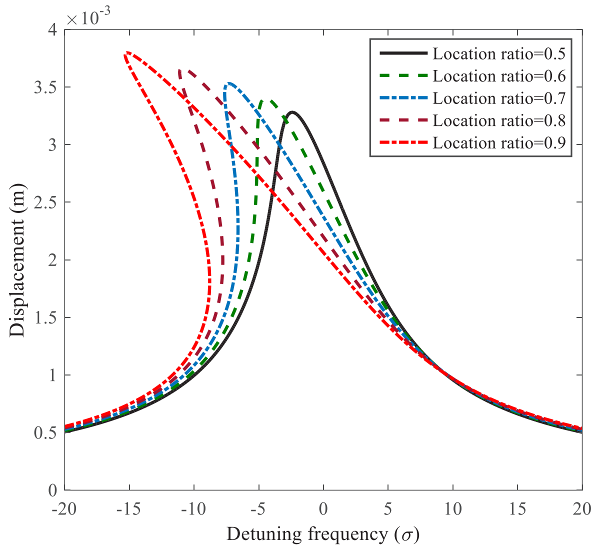

Fig. 4. Frequency-response curves in the primary resonance when location ratio

The following figure shows the increase in the nonlinear coefficient of the quadratic term and the consequent increase in the intensity of the system's nonlinear behavior at different crack depth and location ratios.

3. RESULTS AND DISCUSSION

3.1. The primary resonance response of the cracked beam

The nonlinear vibration of the cracked *beam* is simplified to a *single degree* of *freedom equation as follows:*

$$\frac{d^2}{dt^2}u(t) + 2\varepsilon^2\mu\frac{d}{dt}u(t) + \omega_0^{\,2}u + \varepsilon\beta u^2 = \varepsilon^2 F\cos(\Omega t) \quad \textbf{(5)}$$

The primary resonance of the cracked beam is analyzed by using the method of multiple scales, which results in the following equation:

$$\sigma = \frac{1}{12}\frac{-5a^3\beta^2 m \pm 6(-4a^2m^2\zeta^2\omega_0^{\,8} + f^2\omega_0^{\,4})^{0.5}}{ma\omega_0^{\,3}} \quad \textbf{(6)}$$

Frequency-response curves of the beam for different breathing crack depth ratios are plotted in Fig.3 . The crack location ratio was assumed $\eta = 0.9$. It can be seen from Fig. 3, increasing the crack depth ratio leads to an increase in the intensity of the nonlinearity of the response.

Fig. 4 also compares the frequency-response curves at the primary resonance with the different location ratios of the breathing crack. The crack depth ratio was assumed $\alpha_c = 0.5$.

The effect of damping on the nonlinear response of the cracked beam is shown in Fig. 5. A breathing crack may cause a significant increase in the damping value of the beam [9-11]. As demonstrated in Fig. 5, Increased damping in the cracked beam leads to a decrease in the nonlinearity of the vibrational response.

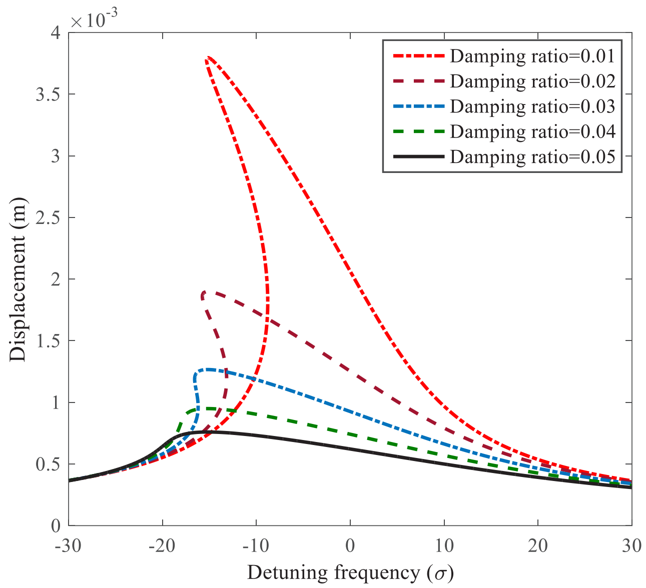

Fig. 5. The effect of damping on the frequency-response curves when $\alpha_c = 0.5$ and $\eta = 0.9$

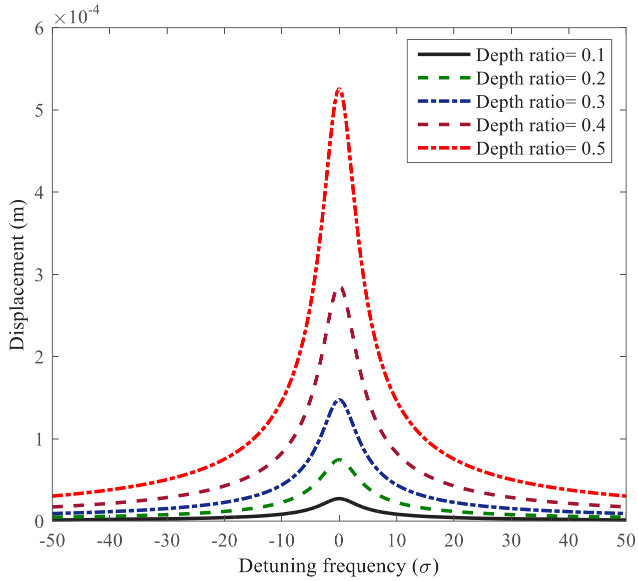

Fig. 6. The effect of crack depth on superharmonic resonance response when $\Omega = (\omega_0 / 2)$ and $\eta = 0.9$

3.2. Superharmonic resonance response of the cracked beam

When Ω is away from the natural frequency of the cracked beam, the effect of the excitation force will be smaller than the resonance case, and the hard excitation equation for the application of the method of multiple scales is written as follows [12]:

$$\frac{d^2}{dt^2}u(t) + 2\varepsilon\mu\frac{d}{dt}u(t) + \omega_0^2 u + \varepsilon\beta u^2 = \frac{f}{m}\cos(\Omega t) \quad (7)$$

The superharmonic resonance of the cracked beam is analyzed by using the method of multiple scales when $\Omega = (\omega_0 / 2)$ which results in the following equation:

$$\sigma = \frac{1}{4}\frac{\sqrt{\frac{f^4\beta^2}{m^4(\omega_0^2 - \Omega^2)^4} - 16a^2\mu^2\omega_0^2}}{a\omega_0} \quad \mathbf{(8)}$$

Fig. 6 shows the frequency-response curves of the beam for different breathing crack depth ratios when the location ratio is $\eta = 0.9$.

4. CONCLUSIONS

In this paper, the nonlinear equation of a cantilever beam with a transverse breathing crack was derived using the Weierstrass approximation theorem. The *primary* and *superharmonic responses* of the cracked beam were obtained using the method of multiple scales. The results show that the nonlinearity of the response at the superharmonic resonance of order ½ was very sensitive to the presence of a breathing crack, and it was strongly dependent on the crack depth and location ratios. According to the results of this study, if the breathing behavior of a fatigue crack is ignored, it can lead to significant errors in investigating the dynamic behavior of the cracked beam. Finally, The main purpose of this study is to develop an effective model that can be used as an efficient tool to analyze the bilinear behavior of a cantilever beam with a transverse fatigue crack.

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