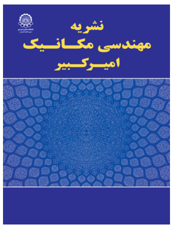

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Investigating The Influence Of Higher-Order Boundary Conditions On Free Vibrations Of Bi-Directional FG Thick Conical Micro-Shells

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ABSTRACT: The present paper investigates the influence of higher-order boundary conditions caused by accounting the small scales effect on free vibrations of bi-directional functionally graded thick conical micro-shells. The present model accounts for the gradation of the material length scale parameter as one of the micro-shell mechanical properties along its thickness as well as its axial axis. The modified couple stress as well as the first-order shear deformable love shell theories together with the Ritz method are employed to obtain the eigenvalue eigenvector equations governing on the free vibrations of the micro-structure. These equations are solved for some different types of boundary conditions. The present findings are compared and successfully validated by the available results in the literature. The influences of small scales, higher-order boundary conditions and power law distribution indices in both the transversal and axial directions on free vibrations of conical micro-shells are then investigated. The results reveal that higher-order boundary conditions play a crucial role in dynamics of conical micro-shells especially when these boundary conditions directly affect the eigenmodes which are dominant in the dynamics of the structure. In addition, it is observed that although the dynamics of the present conical micro-shell is affected by the power law distribution indices in both the transversal and axial directions, it is more sensitive to the transversal one.

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1. INTRODUCTION

Engineering at micron and sub-micron scales is very important nowadays. Bearing in mind that classical continuum mechanics is incapable of describing the behavior of small-scale structures and predicting the behavior of these structures via empirical observations and molecular dynamic simulations is very computational expensive, employing higher-order size-dependent theories of elasticity such as the Modified Couple Stress Theory (MCST) to provide mathematical models for micro-structures motivates many researchers to date.

Despite micro-beams and micro-plates, the number of research works devoted to the analysis of micro-shells based on the MCST is very limited [1]. In addition, amongst all the available literature dealing with the size-dependent investigation of micro-shells, conical structures have been less studied than other types of shells [2]. In this regard, it is worth noting that according to the best of the authors' knowledge, there exists no study in the open literature dealing with the investigation of truncated conical micro-shells made of bi-directional functionally graded materials (FGMs). Therefore, the present work aims to study the size-dependent oscillatory behavior of such structures based on the MCST.

2. MATHEMATICAL MODEL OF THE PROBLEM

According to the MCST, the deviatoric part of the couple stress tensor, conjugated with the symmetric curvature tensor, also acts on material elements of a structure and so, it should be included in the strain energy expression [3]. Considering a truncated conical micro-shell as is shown in Table. 1, the strain energy expression is given by

$$U=\frac{1}{2}\int_{\Omega}\left(\boldsymbol{\sigma}:\boldsymbol{\varepsilon}+\mathbf{m}:\boldsymbol{\chi}\right)d\Omega \tag{1}$$

where $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$, $\mathbf{m}$ and $\boldsymbol{\chi}$ denote Cauchy's stress, strain, deviatoric part of the couple stress, and symmetric curvature tensors. The micro-shell is assumed to be made of a two-phase material graded along the x- and z-directions according to the power-law distribution function. In view of the first-order shear deformable Love shell theory, the displacement field is also assumed to be as

$$\begin{aligned}u(x,\theta,z,t)&=u_0(x,\theta,t)+z\psi_x(x,\theta,t)\\ v(x,\theta,z,t)&=v_0(x,\theta,t)+z\psi_\theta(x,\theta,t)\\ w(x,\theta,z,t)&=w_0(x,\theta,t)\end{aligned} \tag{2}$$

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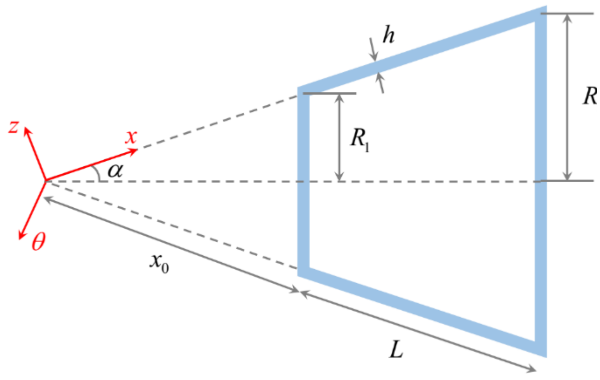


Fig. 1. Side view of a truncated conical micro-shell.

where u_0, v_0 and w_0 denote the displacements of a point placed on the mid-surface and ψ_x and ψ_θ are rotations around the x and θ axes, respectively. Substituting from the strain and kinetic energies into the Hamilton principle, discretizing the displacement components along the x and θ axes, and assuming harmonic motion for the present system, the reduced eigenvalue-eigenfunction equations associated with the present system are obtained as

$$\left([\mathbf{K}]-\omega^2[\mathbf{M}]\right)\{d\}=0 \tag{3}$$

Vanishing the determinant of $[\mathbf{K}]-\omega^2[\mathbf{M}]$, the natural frequencies of the system as well as their corresponding eigenmodes will be obtained.

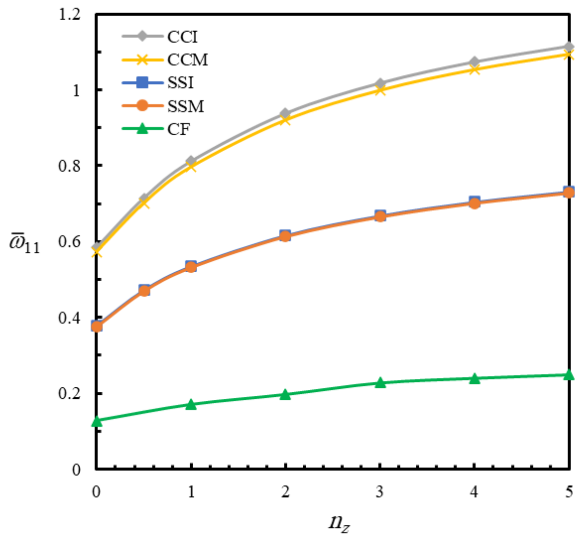
3. RESULTS AND DISCUSSIONS

To investigate the influence of small scales, a homogeneous micro-shell with mechanical properties $E=1.06$ TPa, $\nu=0.3$, $\rho=2300\,\mathrm{kg/m^3}$ and $l=h$ as well as the geometrical properties $\alpha=30^\circ$, $h/R_1=0.1$ and $L/h=50$ is considered.

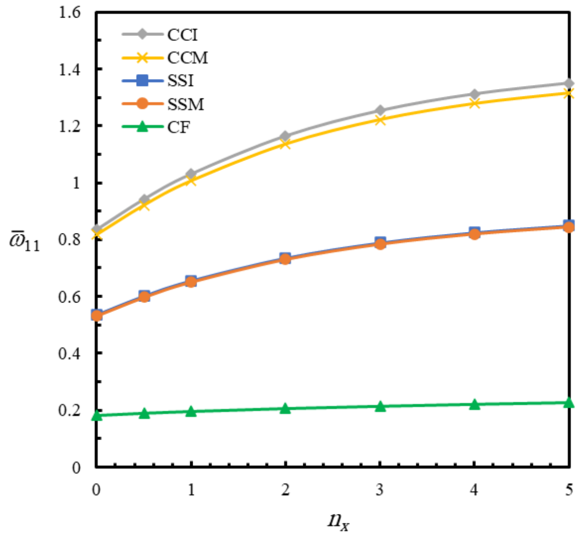
Table 1 compares the normalized size-dependent fundamental natural frequency ($\varpi=\omega R_2\sqrt{\rho\left(1-\nu^2\right)/E}$) of this structure with and without satisfying the higher-order boundary conditions with those obtained based on the CT. As it is seen from this Table, satisfying higher-order boundary conditions satisfying a crucial role in the micro-structure dynamics especially the ones with simply supported boundary conditions.

Table. 2 investigates the influence of the power-law index on the variation of the fundamental natural frequency of the system. To this end, a conical copper-silicon graded micro-shell with geometric properties similar to those selected in the previous section is considered again. The mechanical properties of copper and silicon are assumed to be as $E_{\mathrm{Cu}}=108\,\mathrm{GPa}$, $\nu_{\mathrm{Cu}}=0.32$, $\rho_{\mathrm{Cu}}=8960\,\mathrm{kg/m^3}$, $l_{\mathrm{Cu}}=1.422\,\mu\mathrm{m}$ [4] and $E_{\mathrm{Si}}=169\,\mathrm{GPa}$, $\nu_{\mathrm{Si}}=0.3$, $\rho_{\mathrm{Si}}=2332\,\mathrm{kg/m^3}$ and $l_{\mathrm{Si}}=0.592\,\mu\mathrm{m}$ [5].

It is worth mentioning that by increasing the power-law index along with the micro-shell thickness (i.e. n_z) and length (i.e. n_x), the volume fraction of silicon increases.



(a)



(b)

Fig. 2. Variation of the fundamental natural frequency versus the power law indices

Therefore, in view of the fact that $E_{\mathrm{Si}}>E_{\mathrm{Cu}}$ and $\rho_{\mathrm{Si}}<\rho_{\mathrm{Cu}}$, increasing the power-law indices increases the fundamental natural frequency at all the considered boundary conditions. However, the influence of the power-law index along the micro-shell thickness is more than that of along its length.

4. CONCLUSIONS

This study focused on the investigation of the influence of satisfying the higher-order boundary conditions on free vibration characteristics of thick truncated conical micro-shells made of bi-directional FGM. Despite most of the previous studies, the present work accounted for the gradation of material length scale parameters along with the micro-shell thickness and length. Satisfying all the classical and non-classical higher-order essential boundary conditions, the eigenvalue-eigenfunction problem associated with the present system was solved through the Ritz method. Results revealed

Table 1. Influence of satisfying higher-order boundary conditions on the fundamental natural frequency of the present micro-shell

BCs	CT	MCST-without satisfying higher-order BCs	MCST-with satisfying higher-order BCs
SSM	0.8402	0.8800	1.1511
SSI	0.8664	0.9108	1.1829
CCM	1.2108	1.2253	1.3734
CCI	1.2427	1.2577	1.4010
CF	0.2524	0.2526	0.2635

that satisfying the higher-order boundary conditions seriously affected the natural frequencies and their corresponding mode-shapes of the micro-structure and so plays a crucial role in its dynamic. Especially, when accounting for the influence of the higher-order boundary conditions makes a serious change in the primary variables affecting the transversal stiffness of the structure. It was found that considering the influence of couple stress components increases the natural frequencies of the micro-structure at all types of the studied boundary conditions. In addition, it was observed that increasing the power law indices along both the length and thickness of the micro-shell increases the natural frequencies of the system. However, these changes are more sensitive to the thickness index than the length.

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