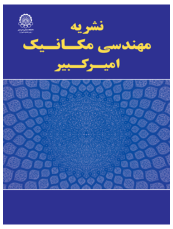

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Lateral Stability Analysis Of Thin-Walled Fiber-Metal Laminate Beam With Varying Cross-Section By Considering Nonlinear Strains

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ABSTRACT: In this paper, the lateral-torsional buckling behavior of thin-walled FML beams with varying I-section is perused using an innovative and accurate methodology. Considering the coupling between the bending displacements and the twist angle, the system of lateral stability equations are derived via energy method in association with Vlasov's model for thin-walled beam and the classic lamination theory. By uncoupling the equilibrium differential equations, the system of governing equations is transformed to a fourth-order differential equation in terms of the twist angle. The differential quadrature method is then applied to solve the resulting equation and to acquire the lateral buckling loads. The accuracy of the proposed methodology has been investigated by comparing the results with the outcomes obtained using ANSYS finite element software. In the following, the effect of significant parameters such as stacking sequence, fiber angle, fiber type, web tapering ratio, load height parameter and volume fraction of metal on lateral buckling load of fixed-free FML tapered I-beam under uniformly distributed load has been investigated. The results shows that the optimum fiber orientation is achieved is obtained by placing fibers at 45 in the web and 0 in both flanges between two aluminum sheets.

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1. INTRODUCTION

Today, the fabrication of thin-walled beams from various materials such as steel, wood, fiber-reinforced composite materials, and functionally graded materials has become possible by developing pultrusion and assembly methods. Fiber metal laminations (FMLs) are a new class of hybrid materials built from several thin sheets of metal alloys and fiber-reinforced epoxy composite plies. In this paper, the lateral-torsional stability analysis of web and/or flanges tapered FML I-beam is investigated using the Differential Quadrature Method (DQM). For this, the general and straightforward procedure suggested by Soltani et al. [1, 2] is adopted.

2. GOVERNING EQUATIONS

A schematic of thin-walled FML beam with length L varying I-section subjected to uniformly distributed load is shown in Fig. 1. The orthogonal right-hand Cartesian coordinate system (x, y, z) is adopted, wherein x denotes the longitudinal axis and y and z are the first and second principal bending axes parallel to the flanges and web, respectively. The origin of these axes (O) is located at the centroid of the cross-section. It is supposed that all section walls are composed of two metal layers at the outer sides of the fiber-reinforced polymer laminates.

Based on small displacements assumption and Vlasov's thin-walled beam theory for non-uniform torsion, the displacement fields can be expressed as [3]:

$$
\begin{aligned}
U(x,y,z) &= u(x) - y\,\frac{dv(x)}{dx} - z\,\frac{dw(x)}{dx} - \omega(y,z)\frac{d\theta(x)}{dx} \\
V(x,y,z) &= v(x) - z\,\theta(x) \\
W(x,y,z) &= w(x) + y\,\theta(x)
\end{aligned}
\tag{1}
$$

where U, V, W stand for to the axial, lateral and vertical displacement components along the x, y, z direction, respectively, whereas u, v, w are the kinematic quantities defined at the reference surface, the term $\omega(y,z)$ refers to the warping function for the variable cross-section, defined by means of Vlasov torsion theory [3], and θ is the twisting angle. In this research, equilibrium equations and boundary conditions are derived from stationary conditions of the total potential energy. Based on this principle, the following relation is obtained

$$
\delta\Pi = \delta U_l + \delta U_0 - \delta W_e = 0 \tag{2}
$$

In this formulation, δ denotes a variational operator. U_l and U_0 represent the elastic strain energy and the strain energy due to effects of the initial stresses, respectively.

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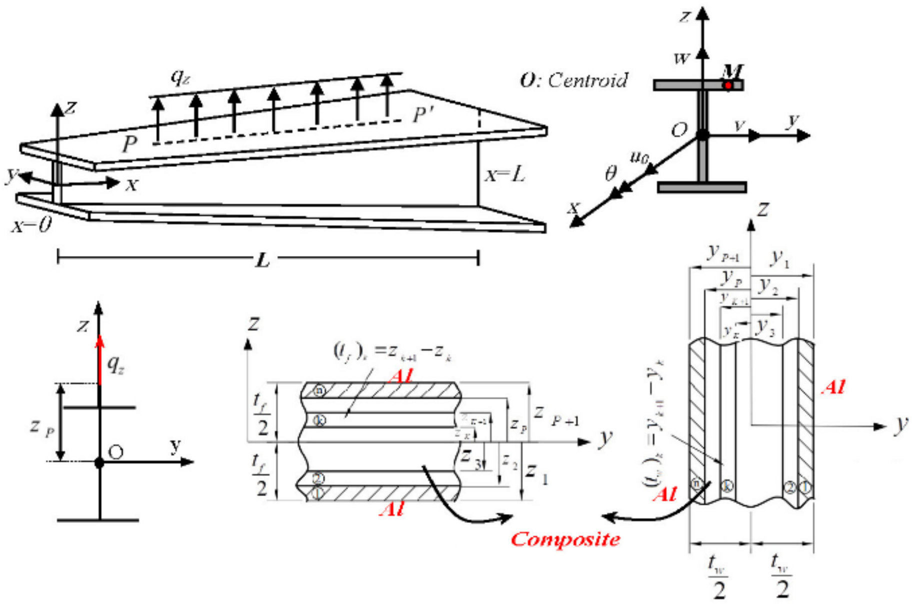

Fig. 1. FML beam with variable doubly symmetric I-section under external distributed loads: Notation for displacement parameters, and web and flanges lay-up arrangement

W_e denotes work done by external applied loads. Their relationships for each term of the total potential energy are developed separately in the following:

$$\delta U_l = \int_0^L \int_A \left(\sigma_{xx}\delta\varepsilon_{xx}^l + \tau_{xy}\delta\gamma_{xy}^l + \tau_{xz}\delta\gamma_{xz}^l\right) dA\,dx \qquad (3a)$$

$$\delta U_0 = \int_0^L \int_A (\sigma_{xx}^0\delta\varepsilon_{xx}^* + \sigma_{xy}^0\delta\gamma_{xy}^* + \sigma_{xz}^0\delta\gamma_{xz}^*) dA\,dx \qquad (3b)$$

$$\delta W_e = \int_0^L q_z \delta w_p dx \qquad (3c)$$

where L and A stand for the beam length and the cross-sectional area, respectively. Moreover, ($\delta\varepsilon_{xx}^i, \delta\gamma_{xz}^i, \delta\gamma_{xy}^i$) and ($\delta\varepsilon_{xx}^*, \delta\gamma_{xz}^*, \delta\gamma_{xy}^*$) refer to the variation of the linear and non-linear part of the strain tensor, respectively; whereas $\sigma_{xx}, \tau_{xz}, \tau_{xy}$ denote the Piola–Kirchhoff stress tensor components, and $\sigma_{xx}^0, \tau_{xz}^0, \tau_{xy}^0$ are the initial axial and shear stress conditions. According to Fig. 1, it is contemplated that the external bending moment occurs about the major principal axis (M_y^*). Therefore, the magnitude of bending moment with respect to z-axis is equal to zero. Regarding this, the most general case of normal and shear stresses associated the external bending moment M_y^* and shear force V_z are considered as:

$$\sigma_{xx}^0 = -\frac{M_y^*}{I_y} z\,;\; \sigma_{xz}^0 = \frac{V_z}{A} = -\frac{M_y^{*\prime}}{A};\; \sigma_{xy}^0 = 0 \qquad (4)$$

In Eq. (3c), w_P is the vertical displacement of point P. According to kinematics used in Asgarian et al. [1] and by adopting the quadratic approximation, the vertical displacement of the point P and its first variation are as follows:

$$w_P = w - z_P\frac{\theta^2}{2} \rightarrow \delta w_P = \delta w - z_P\theta\delta\theta \qquad (5a,b)$$

In this equation, z_P is used to imply the eccentricity of the applied loads from the centroid of the cross-section. The expression of the firs variation of total potential energy is finally obtained as

$$\delta\Pi = \int_L \begin{pmatrix} (EA)_{com} u_0'\delta u_0' + (EI_z)_{com} v''\delta v'' \\ +(EI_y)_{com} w''\delta w'' \\ +(EI_\omega)_{com}\theta''\delta\theta'' + (GJ)_{com}\theta'\delta\theta' \end{pmatrix} dx$$
$$+\int_0^L \left(-M_y^* v''\delta\theta - M_y^*\theta\delta v''\right) dx \qquad (6)$$
$$-\int_0^L \left(q_z\delta w - q_z z_P\theta\delta\theta\right) dx = 0$$

where $(EA)_{com}$ denotes axial rigidity. $(EI_y)_{com}$ and $(EI_z)_{com}$ represent the flexural rigidities of the y- and z-axes, respectively. $(EI_\omega)_{com}$ and $(GJ)_{com}$ are, respectively, warping and torsional rigidities of composite thin-walled beams with doubly symmetric I-section, defined by [4, 5]:

$$\begin{aligned}
(EA)_{com} &= 2b_f A_{11}^f + dA_{11}^w; \\
(EI_z)_{com} &= 2\frac{b_f^3}{12}A_{11}^f + dD_{11}^w \\
(EI_y)_{com} &= 2b_f D_{11}^f + 2\frac{d^2}{4}b_f A_{11}^f + \frac{d^3}{12}A_{11}^w \\
(EI_\omega)_{com} &= 2(\frac{d^2}{4}A_{11}^f + D_{11}^f)\frac{b_f^3}{12} + \frac{d^3}{12}D_{11}^w; \\
(GJ)_{com} &= 4(2b_f D_{66}^f + dD_{66}^w)
\end{aligned} \qquad (7)$$

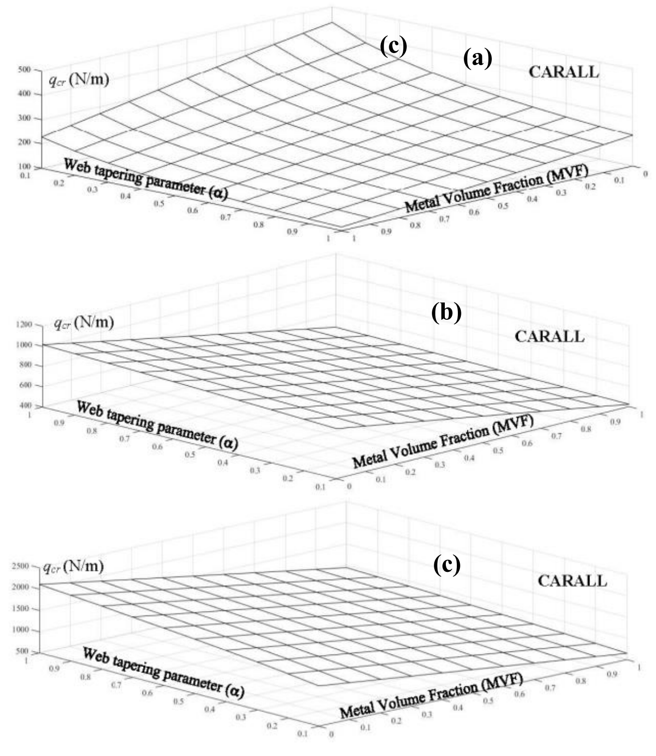

Fig. 2. Variation of the lateral buckling load with respect to the metal volume fraction and web tapering for CARALL section for three different transverse loading positions, (a) Top flange, (b) Centroid, (c) Bottom flange

Based on Eq. (6), the first variation of the total potential energy contains the virtual displacements ($\delta u\ ,\delta v,\delta w,\delta\theta$) and their derivatives. After appropriate integrations by parts, and after mathematical simplifications, we get the following equilibrium equations in the stationary state

$$((EA)_{com}u_0')'=0 \tag{8a}$$

$$\left((EI_y)_{com}w''\right)''=q_z \tag{8b}$$

$$\left((EI_z)_{com}v''\right)''-\left(M_y^*\theta\right)''=0 \tag{8c}$$

$$\left((EI_\omega)_{com}\theta''\right)''-\left((GJ)_{com}\theta'\right)' -M_y^*v''+q_z z_P\theta=0 \tag{8d}$$

In the equilibrium equations, the first and second ones are uncoupled and stable, and do not affect the lateral buckling behavior of I-beam subjected to transverse loading. The differential equations (8c, d) have a coupled nature due to the presence of the lateral deflection v and torsion θ component, as well as the bending moment M_y^*. Based on the straightforward methodology presented by Asgarian et al. [1], the governing equilibrium equation for the torsional angle (8d) can be rewritten as

$$\left((EI_\omega)_{com}\theta''\right)''-\left((GJ)_{com}\theta'\right)'-\frac{M_y^{*2}\theta}{(EI_z)_{com}}+q_z z_P\theta=0 \tag{9}$$

3. RESULTS AND DISCUSSION

To solve the resulting fourth-order differential equation with variable coefficient and to calculate the lateral buckling load of FML web tapered I-beam subjected to different end conditions, the DQM is employed. In order to validate the acquired outcomes of methodology presented herein, comparisons have been carried out with those presented by Soltani et al. [2], and ones obtained by ANSYS software. In this section, the linear lateral buckling analysis is performed for a fixed-free transversely loaded 10-layer FML web tapered I-beam with a span of 10m. All section walls (both flanges and web) are laminated symmetrically concerning its mid-plane and made of Aluminum alloy 2024-T3 (outer metal layers) and Carbon/epoxy material (eight inner composite layers). The material features of the lamina are as follows: for the aluminum plies, E = 72.4 GPa and υ = 0.33; and for the fiber-reinforced composite layers, E_x = 181 GPa, E_y = 10.3 GPa, Gxy = 7.17 GPa, and υxy = 0.28.

Considering the optimum laminate stacking sequence, the lateral-torsional buckling load variation versus the metal volume fraction and web tapering ratio and for three different loading positions is presented in Fig. 2.

4. CONCLUSIONS

The present research deals with the lateral-torsional buckling analysis of FML tapered doubly-symmetric I-beam. Based on Vlasov's theory for thin-walled cross-section and the classical lamination plate theory, the system of two-coupled governing equations is derived via the energy method. The effect of transverse loading position is also considered in the formulation. The differential quadrature method is then used to estimate the buckling load for web and flanges tapered beam. Based on the presented results, the endurable lateral buckling decrease significantly with increasing the volume fraction of the metal. This result is predictable based on the material properties of carbon/epoxy and aluminum. It should be noted that MVF=0 represents full fiber composite I-section in the absence of metal layers, and MVF=1 indicates that all cross-section walls are entirely made of aluminum. Moreover, it is seen that the uniformly transverse load position has a significant effect on the stability strength of unrestrained laminated composite beams with varying doubly-symmetric I-section. For these loading cases, the lateral buckling strength will be improved when the distributed load location is on the bottom flange due to the reduced rotation of the I-section from its original, and the lower values are obtained when the load is applied on the top flange position. Note also that the web non-uniformity parameter has a considerable impact on the lateral-torsional buckling strength.

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