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# Investigating Bi-Stability of Pressurized Piezoelectric Micro-Plates Based on the Modified Couple Stress Theory

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**ABSTRACT:** Recently, it has been substantiated that besides initially curved micro-structures, pressurized flat micro-plates can also experience snap-through instability. Given the potential applications of these micro-plates in designing high-sensitive sensors, the present work aims to investigate the bi-stable behavior of such structures when they are integrated with a piezoelectric layer. To this end, the modified couple stress theory together with the geometric nonlinear Kirchhoff plate model are employed. Hiring Galerkin's method, the reduced governing equilibrium, and stability equations are then achieved. The limit points associated with the micro-plate equilibrium path are then determined through the simultaneous solution of these equations. The present findings are compared and validated by available results in the literature. The influence of the piezoelectric actuation on the bi-stable response of the system is then investigated. The results reveal that the shape of the micro-plate equilibrium path and the number and the position of its limit points can seriously be affected by applying the piezoelectric voltage. Despite the previous studies, the present paper shows that applying positive piezoelectric voltage does not decrease the pull-in threshold of the system all the time and can sometimes increase it when the micro-plate undergoes large differential pressures. Furthermore, the results reveal that applying positive piezoelectric voltages expands the snapping zone while negative ones downsize this region. The present results can be very useful for micro-electromechanical system engineers.

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## 1- Introduction

Investigating the behavior of bi-stable micro-structure as the building block of high-sensitive micro-sensors motivates the attention of many researchers to date [1]. Despite the usual belief that only initially curved micro-structures can experience snap-through instability when they are subjected to loading in the opposite direction of the incline, recently it has been substantiated that pressurized flat systems can also behave bi-stably [2]. Bearing in mind that flat micro-structures are less stiff in comparison to equivalent arch-shaped systems, this strange feature of pressurized flat micro-plates is proposed to be utilized as the operation principle for designing high-sensitive micro-sensors [1, 3].

In view of the fact that micro-plates equipped with piezoelectric layers play a crucial role in designing tunable micro-sensors [4], the main goal of the present work is to investigate the bi-stable behavior of such structures when they exhibit differential pressures. To this end, the governing equilibrium equations are obtained based on the modified couple stress theory. Employing the Galerkin projection method, the governing equilibrium equations are then reduced to some algebraic equations. Vanishing the Jacobean of the reduced governing equations of equilibrium together with these equations themselves, the micro-plate stability is then examined [3]. Hiring the present stability analysis approach, the combined effects of the piezoelectric voltage as well as the differential pressure on the limit points map of the system are studied in detail. The results revealed that despite the negative voltages, applying positive ones expands the bi-stable zone in the limit points map.

## 2- Mathematical Model of the Problem

Fig. 1 depicts a schematic of a pressurized micro-electro-mechanical plate. As it is seen, the length, width, and thickness of the plate are $b$, $a$, and $h$, respectively. The micro-plate is made of two layers: a substrate layer and a piezoelectric one with a thickness of $h_p$. The initial gap between the two electrodes is $d$. It is assumed that the micro-plate is subjected to both the piezoelectric and electrostatic excitations. That is the piezoelectric voltage is $V_p$ and the capacitive voltage, which is applied between the two electrodes, is denoted by $V_{DC}$. The opposing differential pressure between the two electrodes is also $p$. Considering the size-dependent thin plate model based on the modified couple stress theory (MCST) [3], the equilibrium equations are obtained as

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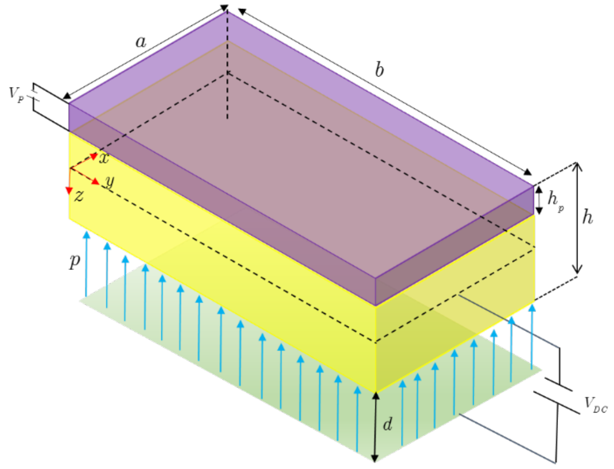


**Fig. 1. Three-dimensional schematic of a rectangular micro-plate with a piezoelectric layer.**

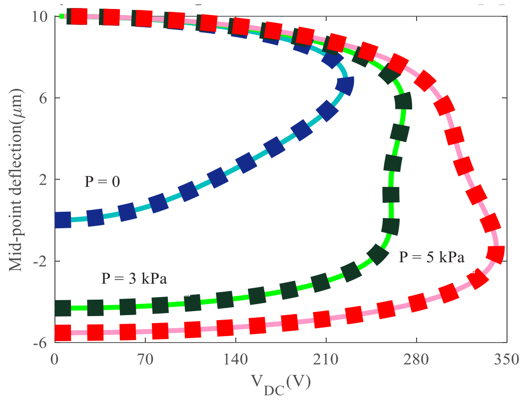


**Fig. 2. Influence of the differential pressure on the behavior of the present silicon micro-plate. Solid lines are corresponding to the present findings and dashed lines depict the results reported in Ref. [3].**

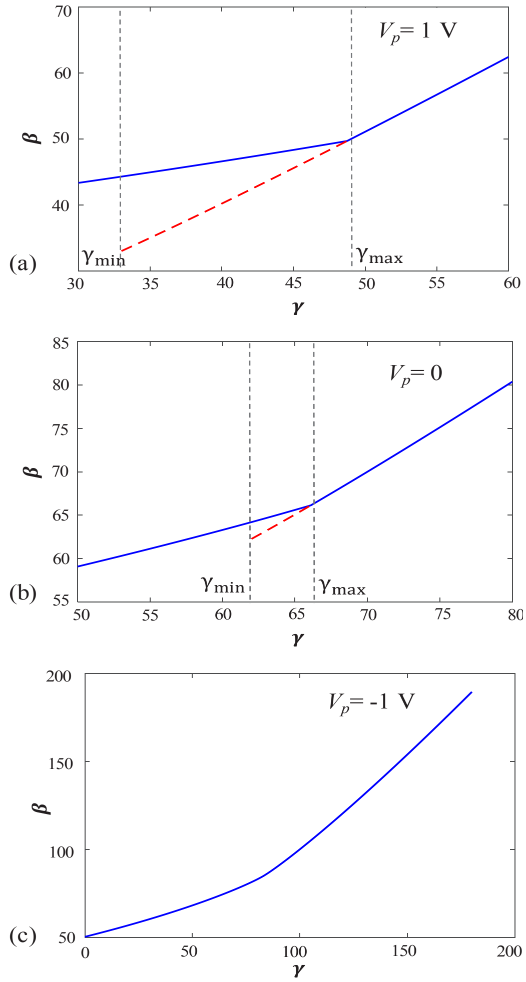


**Fig. 3. Influence of the piezoelectric voltage on the occurrence of snap-through instability**

$$N_{x,x}+N_{xy,y}+\frac{1}{2}\left(\Upsilon_{xz,xy}+\Upsilon_{yz,yy}\right)=0 \tag{1a}$$

$$N_{xy,x}+N_{y,y}-\frac{1}{2}\left(\Upsilon_{xz,xx}+\Upsilon_{yz,xy}\right)=0 \tag{1b}$$

$$\begin{aligned}&\left(N_x w_{,x}+N_{xy} w_{,y}\right)_{,x}+\left(N_{xy} w_{,x}+N_y w_{,y}\right)_{,y}+M_{x,xx}\\&+2M_{xy,xy}+\frac{\varepsilon V_{DC}^2}{2\left(d-w\right)^2}-p+\Upsilon_{xy,xx}-\Upsilon_{xy,yy}\\&+M_{y,yy}+\Upsilon_{y,xy}-\Upsilon_{x,xy}=0\end{aligned} \tag{1c}$$

where, $\varepsilon$ is the dielectric constant of the media between the two electrodes. Also, $N_i, M_i\left(i=x,y,xy\right)$ and $\Upsilon_j\left(j=x,y,xy,xz,yz\right)$ denote the stress and couple stress resultants.

Applying the Galerkin projection method on the in-plane governing equilibrium equations, the in-plane displacements, which are discretized using $n$ basis functions in each direction, can be expressed in terms of the transverse deflection that is approximated with only one basis function. Having the in-plane displacements in terms of the out-of-plane one, the present reduced order model is obtained through the application of the Galerkin method on the transverse equilibrium equation [3]. Differentiating the reduced equilibrium equation with respect to the transverse generalized coordinate, the reduced stability equation can also be obtained [3]. The micro-plate limit points in its equilibrium path can then be obtained through the simultaneous solution of these two equations.

## 3- Results and Discussions

Performing a convergence study, one can simply observe that by setting the number of the included in-plane basis functions in each direction to $n$ = 4, the present solutions are completely converged. To validate the accuracy of the present model, a square silicon micro-plate with $a = 1000\mu m$ , $h = 3\mu m$ , $d = 10\mu m$ , $\nu = 0.3$ , $E = 169\,\text{GPa}$ and $E = 169Gpa$ is considered. The equilibrium paths of this system under some different pressures are compared by those reported in reference [3]. As Fig. 2 depicts, the present solutions agree excellently with those published in the literature [3].

To investigate the influence of piezoelectric actuation, a square silicon micro-plate equipped with a 0.01 µm piezoelectric layer with properties $C_{11}$=132GPa , $C_{12}$=71GPa , $C_{13}$=73GPa , $C_{33}$=115GPa , $e_{13}$=-4.1c.m$^{-2}$ and $e_{33}$=14.1c.m$^{-2}$ is considered. It is also assumed that the initial gap between the two electrodes is three times greater than the micro-plate thickness. Fig. 3 illustrates the limit points map of the present system for three different values of the piezoelectric voltage. As it is seen, applying piezoelectric voltage can seriously affect the bi-stable region in the limit points map graph. Therefore, piezoelectric-based pressurized micro-plates can be considered as the building block for designing tunable high sensitive micro-sensors.

## 4- Conclusions

The main purpose of the present study was to investigate the snap-through instability in pressurized flat micro-plates equipped with a piezoelectric layer. To this end, the modified couple stress thin plate model was considered to obtain the governing equilibrium equations. The Galerkin projection method was then hired to reduce the equilibrium equations to an algebraic equation. Differentiating this algebraic equation with respect to the transverse generalized coordinate, the reduced stability equation was then obtained. Both the stable and unstable behaviors of the system as well as the bi-stable zone in the limit points map were investigated through the simultaneous solution of these two equations. The present findings were compared and successfully validated by the available results in the literature. Despite the previous belief that applying positive piezoelectric voltage decreases the pull-in threshold of the system all the time, it was found that if the system is subjected to some large values of the differential pressure, its pull-in threshold may increase. It was also observed that despite the negative voltages, applying positive ones expands the bi-stable region in the limit points map of the system.

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