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Dynamic Analysis of Micro -Scale Parallelogram Flexures Using Beam Constraint Model and Modified Strain Gradient Theory

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ABSTRACT: In this paper, the dynamic behavior of a small-scale parallelogram (P) flexure is studied. First, using the beam constrain model and the modified strain gradient theory, the nonlinear strain energy of a small-scale beam is obtained in terms of its tip displacements. This energy expression is utilized to derive the strain energy of a P-flexure. Then the governing dynamic equations of motion are derived using Lagrange equations and are linearized around the operating equilibrium point. This linear model is employed to determine the allowable forces which do not lead to instability of the system. Moreover, the natural frequencies of the system are also extracted and the size effect as well as the static components of the applied loads on them are studied in detail. It is observed that by reducing the dimensions, the normalized transverse natural frequency of the system is increased. However, since there is no strain gradient in an axial mode, the axial normalized frequency is remained constant reducing the dimensions of the system. Moreover, it was observed that the tensile static forces lead to an increase, and transverse forces lead to a decrease in normalized natural frequency of the system. The procedure utilized for dynamic modeling of parallelogram flexures in this paper can be further extended for modeling more complex flexure systems.

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1- Introduction

Flexure mechanisms are kind of mechanisms that instead of using classical joints, employs the elastic deformation of its elements for providing the desired motion. Since fabricating classical mechanisms in small dimensions is very difficult, the main advantage of the flexure mechanisms is their application in small scales. Miniaturization provides the possibility of reducing the dimensions of the currently available devices. Moreover, it offers the possibility of fabricating new systems with unique applications. Among various examples of small-scale systems, one can point to the position sensors [1], accelerometers [2], force sensors [3], resonators [4], pressure gauges [5] gyroscopes [6], and micromirrors [7]. Most of the micro-scale mechanisms are fabricated using small-scale parallelogram (P) flexures which in turn are made up of two slender parallel beams connected to a moving stage. This element has very large stiffness in the constraint (rotational and axial) directions while presenting very low stiffness in the transverse direction. This specification has made them very suitable for applications in positioning systems. So, dynamic analysis of these elements is of primary importance.

The beam constraint model (BCM) is a simple yet efficient approach for the analysis of flexure mechanisms. The base of this method was first presented by Awtar and Slocum [8]. Then Awtar and Sen [9, 10] extended this method by presenting a closed-form expression for the nonlinear strain energy of a beam in terms of its tip displacements. This innovation made this method very suitable for the analysis of more complex flexure units. Based on BCM, the static [11-13] and vibration [14-16] behavior of many flexure systems were studied. However, all these researches were based on classical elasticity theory which is not sufficiently accurate in small-scale systems. So, the objective of the current research is to extend the BCM to micro dimensions using the modified strain gradient method and then use it for dynamic analysis of micro-scale P-flexures.

2- Methodology

Fig. 1, shows the schematic view of a P-flexure with a rigid motion stage, under the effect of end loads F_X, F_Z and M_Y. The length, width, and thickness of the beams are respectively L, b, and h.

In P-flexures, the rotation of the stage is very small and can be easily neglected [16]. In this condition, the transverse and axial displacement components of the beams will be identical to those of point O. Moreover, it can be shown that the normalized nonlinear strain energy of a P-flexure using BCM and modified strain gradient theory can be obtained as

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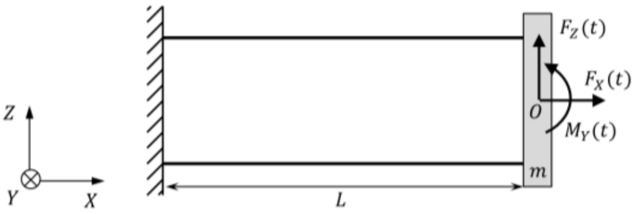

Fig. 1. A micro-scale P-flexure

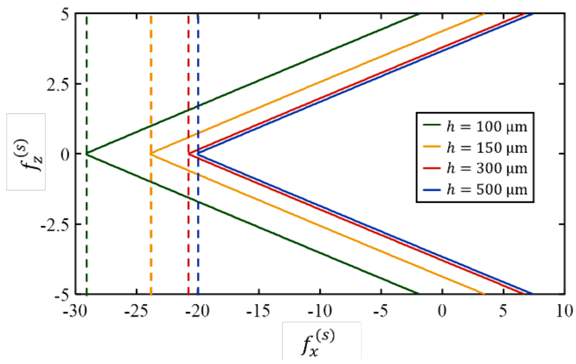

Fig. 2. Stability map and the w<0.15 limit for a P-flexure assuming l=17.6 µm

$$v_P = a_1 \frac{\left(u\left(\hat{t}\right)+\frac{1}{2}k_{11}^{(1)}w^2\left(\hat{t}\right)\right)^2}{1-a_1k_{11}^{(2)}w^2\left(\hat{t}\right)} + k_{11}^{(0)}w^2\left(\hat{t}\right) \tag{1}$$

where u and w are respectively the normalized axial and transverse displacements of O and $\hat{t}$ is the normalized time. Additionally, a_1, $k_{11}^{(0)}, k_{11}^{(1)}$ and $k_{11}^{(2)}$ are some constants that depend on the geometry of the system. Using a dimensional form of Eq. (1) as the potential and $T_P = mL^2\left((du/dt)^2+(dw/dt)^2\right)/2$ as the kinetic energy of the system, Lagrange equations can be employed to derive the normalized equations of motion as

$$\ddot{u} + \frac{2u + k_{11}^{(1)}w^2}{1/a_1 - k_{11}^{(2)}w^2} = f_x \tag{2}$$

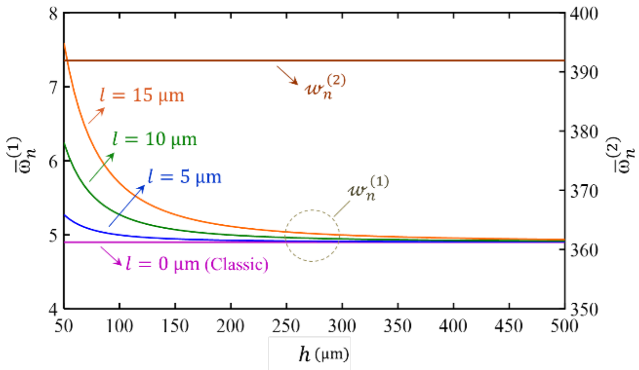

Fig. 3. Natural frequencies of a micro-scale P-flexure

$$\ddot{w} + \left(2k_{11}^{(0)} + k_{11}^{(1)}\left(\frac{2u + k_{11}^{(1)}w^2}{1/a_1 - k_{11}^{(2)}w^2}\right) + k_{11}^{(2)}\left(\frac{2u + k_{11}^{(1)}w^2}{1/a_1 - k_{11}^{(2)}w^2}\right)^2\right)w = f_z \tag{3}$$

where f_x and f_z are respectively the normalized forms of F_X and F_Z shown in Fig. 1., Eqs. (2) and (3) can be linearized around the operating point as

$$[\mathcal{M}]\begin{Bmatrix}\ddot{\hat{u}}\\ \ddot{\hat{w}}\end{Bmatrix} + [\mathcal{K}]\begin{Bmatrix}\hat{u}\\ \hat{w}\end{Bmatrix} = \begin{Bmatrix}\hat{f}_x\\ \hat{f}_z\end{Bmatrix} \tag{4}$$

where the mass matrix $[\mathcal{M}]$ is the identity and $[\mathcal{K}]$ is the symmetric stiffness matrix. By observing the location of the poles of the transfer function of the system, its stability limit can be determined.

3- Results and Discussion

In Fig. 2, the stability limit along with the map of $w<0.15$ within which the BCM is valid, is depicted for a P-flexure for the case of $l=17.6\ \mu$m. Also, the natural frequencies of the system and their dependence on h are depicted in Fig. 3. It is observed that as h is increased, the results of the proposed model tend to be those of classical BCM. However, at small dimensions, there is a remarkable deviation between the models.

4- Conclusion

The objective of the current research is an analysis of the dynamic behavior of P-flexures as the flexure module utilized in most compliant mechanisms. To this end, first, the nonlinear strain energy of the P-flexure is determined using the BCM and modified strain gradient theory. Then, Lagrange equations were employed to determine the governing equations of motion. The linearized form of these equations was employed to determine the stability map of the system and to study the corresponded eigenvalue problem. The related results were used to study the effects of the applied static loads as well as the dimensions of the system on the system. The approach proposed in this paper can be further extended to study the dynamic behavior of more complex compliant mechanisms.

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