Development of a Reduced Order Model of Geostrophic Flow based on a Combination of Proper Orthogonal Decomposition and Long-Short Term Memory Network

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ABSTRACT

Mathematical modeling is used to study the phenomena and behavior of the system. Complex mathematical equations require powerful and time-consuming computational tools where that must be examined in order to obtain the correct behavior of a system. However, they require robust computational tools and take a lot of time. High-accuracy numerical simulations utilize numerical schemes and modeling tools to solve this set of equations and generate useful information about the behavior of a system. It makes many restrictions on the solution of scientific problems in different research fields such as geophysical and atmospheric flows, which have high temporal and spatial variations. Therefore, the development of effective and robust algorithms to achieve the maximum quality of numerical simulations with the optimal computational cost is a research topic. There are several methods for dimension reduction but this study used a combination of Proper Orthogonal Decomposition and long-short term memory network. Finally, comparing the results related to the modal coefficients which are obtained by the reduced order model and CFD snapshots projection shows the high accuracy of the proposed method. Also, one of the items considered in the study of algorithms is the time complexity of the algorithm. The computational time of the proposed method which is reconstructed using 15 modes is ten times faster than when all features have been used to reconstruct the model.

KEYWORDS

Proper Orthogonal Decomposition, Long-Short Term Memory network, Reduced Order Model, Geophysical data

1. Introduction

In general, the reduced-order modeling is a method that replaces the original model of the system with a model, which has low dimension. So that it can retain some features of the system as well [1]. In the desired model, the reduced-order model should have dimensions that are close to the minimum number of parameters necessary to explain the behavior of the system. Many existing reduced-order modeling algorithms have been developed for linear systems. All these approaches are projection methods that convert the high-dimensional space of the original model into a low-dimensional subspace [2]. There are different model order reduction schemes in linear and nonlinear systems. In nonlinear systems, model order reduction is a more difficult problem. Therefore, it is the research topic in many studies today. Some of these studies include inertial manifold approximation, center manifold theory, nonlinear normal modes (NNMs), and POD[1][3]. The proper orthogonal decomposition is an efficient method for extracting reduced basis functions for many physical systems [4]. Nowadays, the combination of machine learning and deep learning methods with traditional methods is used to create a reduced-order model with higher accuracy and a wide range of validity. The main goal of this research is to develop a reduced-order model as a surrogate model for predicting the geophysical (oceanic) data based on a hybrid framework.

2. Methodology

In this research, a hybrid framework using the proper orthogonal decomposition method and the LSTM[2] network is presented. First, we used the proper orthogonal decomposition to calculate modes and modal coefficients of the snapshots and then the model order reduction has been performed according to the number of selected modes. According to each mode, there is a vector of temporal coefficients that moves on this vector using the sliding window method to generate the input-output data required for training the long short-term memory network. The end part of each vector is considered as test data that was not in the training data set. Figure 1 shows the scheme implemented for generating the training dataset. In this figure at first, the POD is used for reducing low dimension subspace and then the LSTM network is used for predicting coefficients. In this research, the mean absolute scaled error (MASE) metric was used to determine the difference between the predicted values and expected values [5]. This error is calculated using equation (1). As can be seen, in the numerator of this relationship, the quantity *e*, which is the prediction error and is obtained from the difference of the predicted value from the original.

$$MASE = mean\left(\left|\frac{e_t}{\frac{1}{n-1}\sum_{i=2}^{n}\left|Y_i - Y_{i-1}\right|}\right|\right) \qquad (1)$$

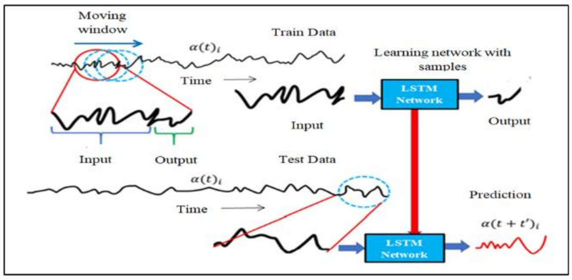
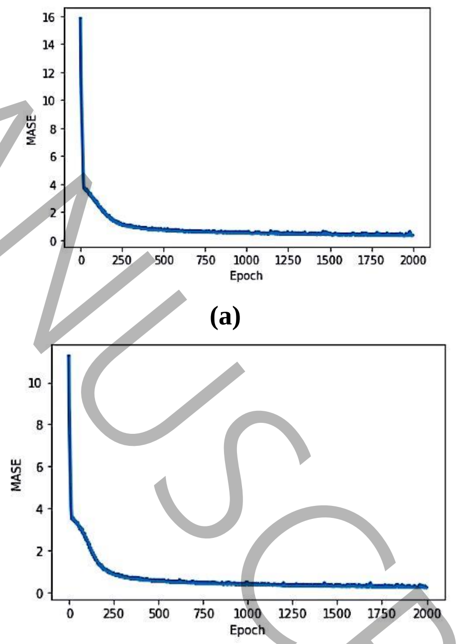


Figure 1. Training strategy of LSTM networks with the input-output framework.

Figure 2 shows the MASE metric for LSTM prediction for all samples using 10 and 15 modes. The results show that the error is generally low, and as can be seen in this figure by increasing the number of modes, the initial error has also decreased. In this figure, the vertical axis indicates the amount of error and the horizontal axis indicates the execution periods of the long-short term memory network.



(a)

(b)

Figure 2.MASE for the LSTM prediction on all samples by using (a) 10 modes and (b) 15 modes

3. Discussion and Results

In order to implement the reduced-order model, Python version 7.3 has been used in the PyCharm environment on a computer with 4 GB memory and a 2 GHz tri-core processor. The data set used in this research includes 90 snapshots on a 150×300 grid and in a domain with

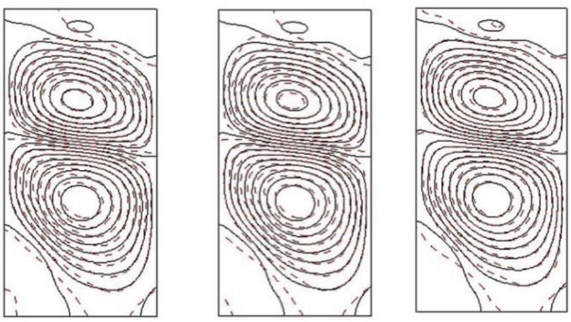
[1] Proper Orthogonal Decomposition
[2] Long-Short Term Memory

dimension of 1×2, where the y-coordinate increases towards the north and the x-coordinate increases towards the east. The flow field is the type of Near-surface oceanic flow. All the numerical simulations in this research have been done for the dimensionless time of 3.78 and dimensioned time of 2.17 years. Also, the snapshots used in the dimensionless time period of 2.88 to 3.78 have been created and used to create a proper orthogonal decomposition basis. Then, the temporal coefficients obtained from the decomposition of the snapshot have been used to train the long short-term memory network. For this purpose, 7 modes are extracted from the data, then the network is trained on each of the seven time-dependent modal coefficients. Table 1 shows the reconstruction error of the vorticity and stream-function field by the hybrid model using 7, 10 and 15 modes. As can be seen in Table 1, the amount of error between the original and reconstructed data is small, and with the increase in the number of modes, this value decreases further.

Table 1. Reconstruction error of the streamfunction and vorticity fields

Quantity	7 modes	10 modes	15 modes
Vorticity	1.022E-02	8.126E-03	5.9165E-03
Stream function	1.515E-01	5.978E-02	2.0934E-02

Figure 3 demonstrates the reconstruction of the last snapshot of the streamfunction field using 7, 10 and 15 modes. As shown in this figure, the results of the model using 7 modes have more errors and as the number of modes increases, the result becomes more accurate. A comparison was made between time evolution of two more energetic modal coefficients of the vorticity field, are shown in Fig. 4.

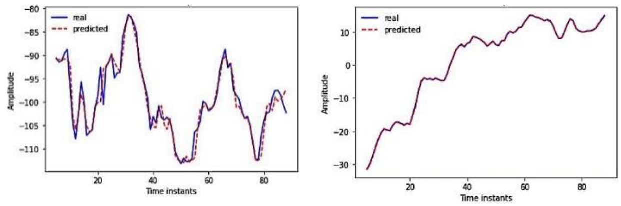


(a) (b) (c)

Figure 3. Comparison of the last original data (black solid lines) and the last snapshot of streamfunction field reconstructed by the reduced order model (red dashed lines) (a) reconstruction using 7 modes, (b) using 10 modes and (c) using 15 modes

4. Conclusions

In this research, a reduced-order model based on the combination of proper orthogonal decomposition and deep learning algorithm specifically the long short-term memory network is presented. The prediction of the temporal coefficients confirms that the proposed model while having high and appropriate accuracy can significantly reduce the calculation time. In terms of time complexity, generating snapshots (even with small and medium numbers) using traditional method requires significant time. For example, it takes about 6 hours to produce 90 snapshots with a personal computing system. While using the method proposed in this research, it takes about 15 minutes with the same system, which shows a significant reduction in the calculation time. According to the estimates, the time required for the simulation based on computational fluid dynamics for the range considered as a test (model prediction) is 30,000 times more than the proposed method.



(a) (b)

Figure 4. Time evolution of the two more energetic modal coefficients of the vorticity field, The original data(black lines) and the reduced order model using 30% of the data for training (red lines), (a) the first mode, (b) the second mode, (c) the third mode

5. References

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