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Nonlinear Torsional Vibrations of Axially Loaded Pretwisted Beam with Primary Resonance Excitations

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ABSTRACT: Frequently used thin walled beams have low torsional stiffness and their torsional deformations may be of such magnitudes that it is not adequate to treat the angles of cross section rotation as small. In this paper, nonlinear torsional vibrations of thin walled beams will be investigated. The method of multiple scales will be implemented as a solution method and different nonlinear phenomena will be studied. The obtained results are compared with the available results in the literature which reveals an excellent agreement between different solution methodologies. The outcomes of this study show that beam nonlinear torsional dynamics and the related phenomena could influence the linear torsional dynamic of beams under axial load, e.g. rotating beams. Forced torsional vibrations of a beam with the excitation in the form of primary resonance of the first and second modes have been investigated. It has been demonstrated that in the case of the beam with two ends clamped boundary conditions, three-to-one internal resonance will appear. The primary resonance of the first and second modes has been solved in two sets of boundary conditions, torsionally clamped-fixed and torsionally fixed-fixed. Nonlinear response, amplitude-phase equations, fixed points, and their stability have been studied.

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1- Introduction

Nonlinear torsional vibrations of the beam have great practical importance in advanced engineering structures. It is shown in the literature (e.g. Ref. [1-3]) that the collective effect of axial load and pretwist angle leads to some interesting phenomena in torsional vibrations of beam-like structures. Nonlinear torsional vibrations of pretwisted axially loaded thin walled beam under primary resonance excitation of the first and second torsional modes will be investigated in this paper.

Methodology

The problem of nonlinear torsional vibrations of a beam with primary resonance excitation is addressed in the following context:

There is not any material or geometric coupling between the beam's bending and torsional degrees of freedom,

Only axial and torsional degrees of freedom are retained in the displacement field,

The beam will undergo large torsional motions

Primary and secondary warpings in conjunction with warping inertia are considered in the model,

Hamilton's principle is implemented in order to extract the governing equations of motion and the corresponding boundary conditions,

The governing equations of motion are cast into one equation neglecting axial inertia,

It is shown that the collective effect of pretwist angle and axial load leads to static untwist.

Primary resonance of the first and second modes is considered.

Fig. 1 shows the thin walled beam with pretwist angle which is considered in this paper.

The stain field could be stated as follows [1],

$$\begin{aligned}
&\varepsilon_{zz}(n,s,z,t)=\varepsilon_{zz}^{0}(s,z,t)+n\varepsilon_{zz}^{n}(s,z,t),\\
&\varepsilon_{zz}^{0}(s,z,t)=w_0'-\phi''(z,t)F(s)+\frac{1}{2}\left(\underline{\phi'^2}+2\beta'\phi'\right)\left(x_p^2+y_p^2\right),\\
&\varepsilon_{zz}^{n}(s,z,t)=r_t(s,z)\phi''(z,t),\\
&\gamma_{sz}(s,z,t)=\Psi\phi'(z,t).
\end{aligned} \tag{1}$$

The nonlinear governing differential equation of motion is as follows:

$$\ddot{\phi}-\phi''+\frac{k_4}{k_1L^2}\phi''''=\frac{k_2}{k_1L}\phi'\phi''+\frac{k_3}{k_1L^2}\phi'^2\phi''+F\cos\Omega\hat{t}. \tag{2}$$

Utilizing the method of multiple time scales as [4],

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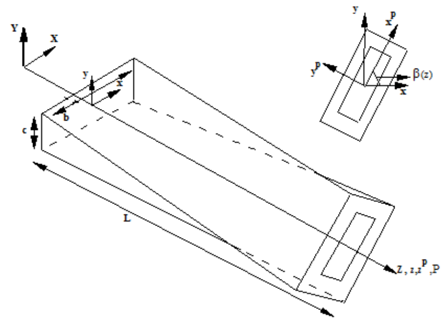

Fig. 1. Schematic of the beam with pretwist angle

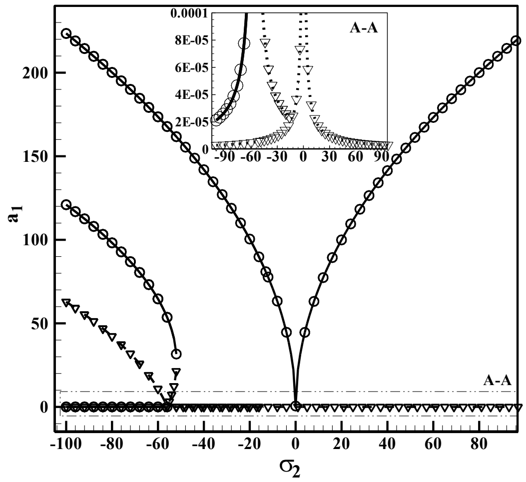

Fig. 2. First torsional mode amplitude versus frequency for the primary resonance of the first mode of the torsionally clamped-free beam, solid line with circle represent stable node; dashed line with triangle represent unstable saddle.

$$\phi(z,t)=\varepsilon\phi_1(\hat{z},T_0,T_2)+\varepsilon^2\phi_2(\hat{z},T_0,T_2)+\varepsilon^3\phi_3(\hat{z},T_0,T_2)+\dots \tag{3}$$

The governing equations will be obtained as,

$$\begin{aligned}
&D_0^2\phi_1-\phi_1''+\frac{k_4}{k_1L^2}\phi_1''''=0,\\
&D_0^2\phi_2-\phi_2''+\frac{k_4}{k_1L^2}\phi_2''''=\frac{k_2}{k_1L}\phi_1'\phi_1''+\delta_s f\cos\Omega T_0,\\
&D_0^2\phi_3-\phi_3''+\frac{k_4}{k_1L^2}\phi_3''''=-2D_0D_2\phi_1+\frac{k_2}{k_1L}\left(\phi_1'\phi_2''+\phi_1''\phi_2'\right)\\
&\qquad+\frac{k_3}{k_1L^2}\phi_1'^2\phi_1''+\delta_p f\cos\Omega T_0.
\end{aligned} \tag{4}$$

The corresponding parameters are defined in the paper. The primary resonances of first and second modes have been solved in two sets of boundary conditions, torsionally clamped-fixed and torsionally fixed-fixed. Nonlinear response, amplitude-phase equations, fixed points, and their stability have been studied. The effect of internal resonance is also addressed.

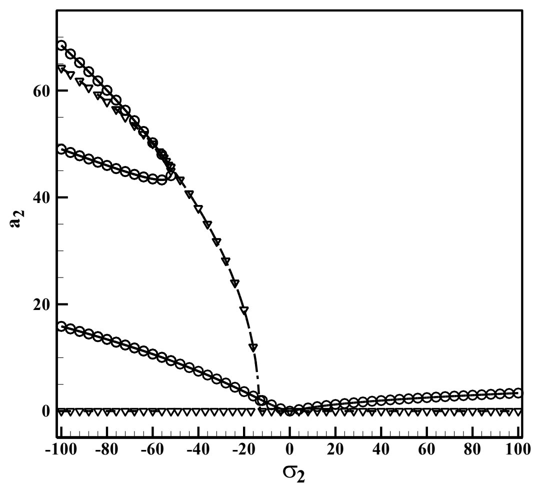

Fig. 3. Second torsional mode amplitude versus frequency for the primary resonance of the first mode of the torsionally clamped-free beam, solid line with circle represent stable node; dashed line with triangle represent unstable saddle.

2- Results and Discussion

The paper contains various results concerning the primary resonance of first and second torsional modes with/without internal resonance in both torsionally fixed-fixed and fixed-free boundary conditions. The linear vibration problem in various boundary conditions is also addressed. Figs. 2 and 3 show the amplitude of the first and second torsional modes of the clamped-free beam in primary resonance conditions. The geometric and material properties of the beam are represented in the full paper.

The obtained results show that the problem consists of multiple fixed points for different values of detuning parameters.

It should be noted that clamped-free torsional boundary conditions lead to a 3 to 1 internal resonance between first and second torsional modes.

Fig. 4 shows the amplitude-frequency plot of the first torsional mode for the torsionally clamped-clamped under primary resonance excitation of the first mode. Geometric and material properties of the beam are represented in the full paper and for the sake of brevity not included here.

3- Conclusion

Nonlinear forced torsional vibration of pretwisted beam under axial loading is considered. Primary resonance of first and second torsional modes is considered. The structural model incorporates a number of non-classical effects such as restrained warping, trapeze effect, and warping inertia.

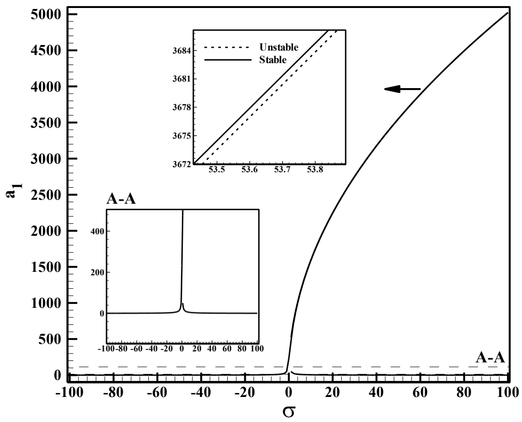

Fig. 4. first torsional mode amplitude versus frequency for the primary resonance of the first mode of torsionally clamped-clamped beam.

Nonlinear equations of motion are derived using Hamilton's principle and are solved using the method of multiple scales. A number of conclusions are outlined as,

The cumulative effect of axial loading and pretwist angle leads to an untwist phenomenon which is demonstrated and validated against existing results.

Linear torsional vibrations of the beam in different boundary conditions have been investigated.

Torsional vibration of the beam with clamped-free torsional boundary condition consists of 3 to 1 internal resonance between first and second torsional modes.

Internal resonance acts as a mechanism for the transfer of energy between vibration modes.

System response at resonance condition will consist of both natural frequencies of first and second modes and some other harmonics. The latter ones stem from the pretwist angle and axial loading and consist of 2×first natural frequency, 2×second natural frequency, and summation and extraction of first and second natural frequencies.

It is shown that the nonlinear Wagner term has a stiffening nature while the trapeze effect has to soften effect I beam's torsional vibrations.

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