



## Theoretical Analysis of the Effects of Hardening Laws, Normal and Through Thickness Shear Stresses on Forming Limit Curves of AA6016-T4

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**ABSTRACT:** Forming limit Curves are one of the common tools to predict the necking in various forming processes. In this study, the Marciniak-Kuczynski instability theory by applying the Gotoh yield function is utilized to estimate the forming limit curves for the AA6016-T4 aluminum sheet in plane stress conditions. Also, the effect of three different hardening models including Swift, Voce, and a linear combination of the Swift and Voce models to determine the limit curves are investigated. The comparison between the theoretical forming limit curves and experimental results from the Nakajima test determines the accuracy of the hardening models in predicting the limit strains. Since in many new forming processes such as hydroforming and incremental sheet forming processes, investigation of the process in plane stress state is not an exact assumption, Therefore, in continuation of the paper, generalized forming limit curves are plotted based on the developed Marciniak-Kuczynski model by extending the Gotoh yield function, and the effect of compressive normal stress and through-thickness shear stress on forming limits of the sheet are investigated. The results indicated that by applying the compressive normal stress and through-thickness shear stresses, the limit strains increase, and the formability is improved, in contrast, limit stresses move down in the diagram.

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### 1- Introduction

Although plane stress condition is an acceptable assumption to determine the forming limit curve in a lot of metal forming processes, in some industrial applications such as hydroforming and incremental sheet forming processes, the normal stress and shear stress effects should be considered. Banabic and Soare [1] extended the Marciniak-Kuczynski (M-K) instability model to investigate the influences of the normal stress on Forming Limit Diagram (FLD). According to their calculations, by increasing the through-thickness normal stress the formability of sheet metal improves. Alwood and Shouler [2] proposed the generalized forming limit diagram and they proved that both compressive normal and through-thickness shear stresses enhance the forming limit strains. Nasiri et al. [3] analyzed the effect of compressive normal stress and through-thickness shear stress on forming limit diagrams of AA3104-H19 alloy based on different yield functions. It was found that the effect of normal stress on limit strains are more than shear stress. Sojodi et al. [4] developed the M-K instability theory model to investigate the effect of normal stress. The 3D stress state was changed to a plane-stress condition based on the hypothesis that hydrostatic pressure does not have any influence on plastic deformation. In this paper, by applying the Gotoh yield function, the effects of different hardening models on limit strains for AA6016-T4

are investigated. Also, the M-K instability model is extended to determine the influences of the through-thickness normal and shear stresses on forming limit diagrams.

### 2- Constitutive Model

In this study, the Gotoh yield function is used to predict the yield behavior, and Swift, Voce, and a linear combination of Swift and Voce models (LSV) are selected to describe the plastic behavior of the AA6016-T4 aluminum alloy.

#### 2- 1- Yield functions

Gotoh yield criteria are expressed as:

Where  $A_{1-9}$  are the constant parameters for the Gotoh yield function [5].

$$\bar{\sigma}^4 = A_1\sigma_1^4 + A_2\sigma_1^3\sigma_2 + A_3\sigma_1^2\sigma_2^2 + A_4\sigma_1\sigma_2^3 + A_5\sigma_2^4 + (A_6\sigma_1^2 + A_7\sigma_1\sigma_2 + A_8\sigma_2^2)\sigma_{12}^2 + A_9\sigma_{12}^4 \quad (1)$$

#### 2- 2- Work-hardening models

To investigate the influence of the hardening law on the forming limit diagrams, different hardening models are utilized to describe the mechanical behavior of the AA6016-T4 alloy.

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Swift hardening model:

$$\bar{\sigma} = S_1 (\bar{\varepsilon} + \varepsilon_0)^{S_2} \quad (2)$$

$$\bar{\sigma} = \sigma_0 + V_1 (1 - e^{-V_2 \bar{\varepsilon}}) \quad (3)$$

$$\bar{\sigma} = X \left( S_1 (\bar{\varepsilon} + \varepsilon_0)^{S_2} \right) + (1 - X) \left( \sigma_0 + V_1 (1 - e^{-V_2 \bar{\varepsilon}}) \right) \quad (4)$$

Where  $S_1, S_2, V_1, V_2$  and  $X$  are the constant parameters for different hardening models [6].

### 3- Marciniak-Kuczynski Model

The M-K theory is one of the most powerful instability theories to determine the onset of localized deformation.

#### 3- 1- M-K model under plane stress condition

This method is based on the existence of the initial imperfection that is characterized by the reduction of thickness in a part of the sheet. In the M-K approach, the equivalent strain increment  $d\bar{\varepsilon}$  with a specific stress ratio ( $\alpha = \sigma_2 / \sigma_1$ ) was applied to the safe region and then the other strain and stress component values in this area were computed by using the flow rule, hardening equation, and yield function. The unknown parameters in the groove zone were calculated according to three major assumptions including compatibility condition, geometrical imperfection, and force equilibrium. The Numerical Newton-Raphson method is used to solve the nonlinear set of equations, and the unknown stress and strain components in the defect region are obtained when the effective strain increment in the groove reaches ten times greater than the perfect area. This numerical procedure in

each stress ratio is repeated for different groove directions to determine minimum limit strains [7].

#### 3- 2- Developed M-K model

To solve the M-K model with normal and shear stresses, the 3D stress state should be converted to a 2D stress state based on the principle that the hydrostatic pressure does not affect the plastic deformation [4].

$$P_4^{3D}(\sigma_1, \sigma_2, \sigma_3, \sigma_{12}, \sigma_{23}, \sigma_{13}) = P_3^{2D}\left(\sigma_1 - \sigma_3, \sigma_2 - \sigma_3, \sqrt{\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2}\right) \quad (5)$$

Based on the flow rule, the corresponding strain increments of the safe region are obtained [4]:

$$d\varepsilon_1^a = d\bar{\varepsilon}^a \frac{\partial \bar{\sigma}^a}{\partial (\sigma_1^a - \sigma_3^a)} \quad (6)$$

$$d\varepsilon_2^a = d\bar{\varepsilon}^a \frac{\partial \bar{\sigma}^a}{\partial (\sigma_2^a - \sigma_3^a)} \quad (7)$$

To determine the strain and stress components in the defect region, the below equations must be considered:

$$\sigma_3^b = \sigma_3^a, \quad \sigma_{t3}^b = \sigma_{t3}^a, \quad \sigma_{n3}^b = \sigma_{n3}^a / f \quad (8)$$

the unknown parameters in the defect area are determined according to the explanations in the previous section [3].

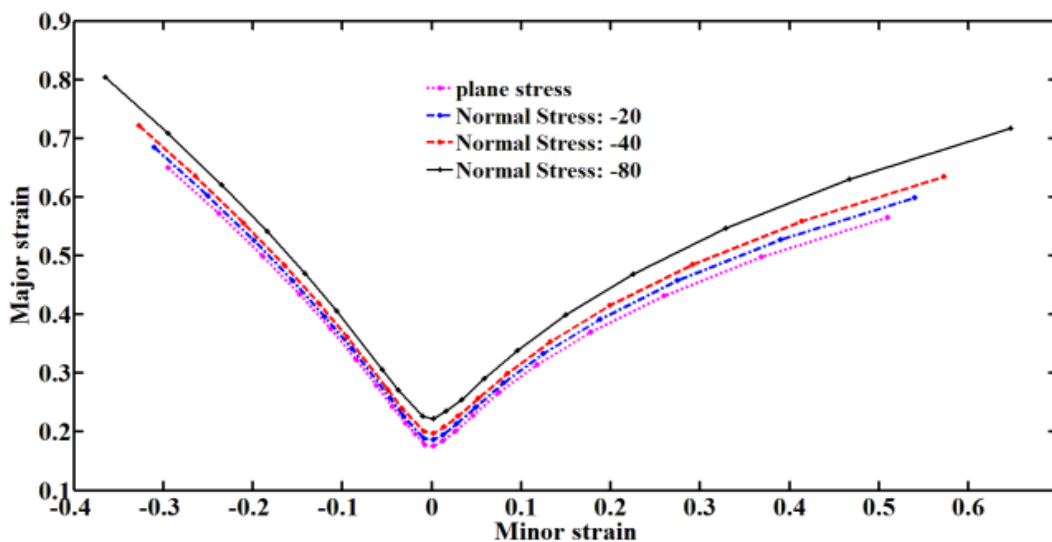


Fig. 1. Forming limit diagrams by considering the compressive normal stress

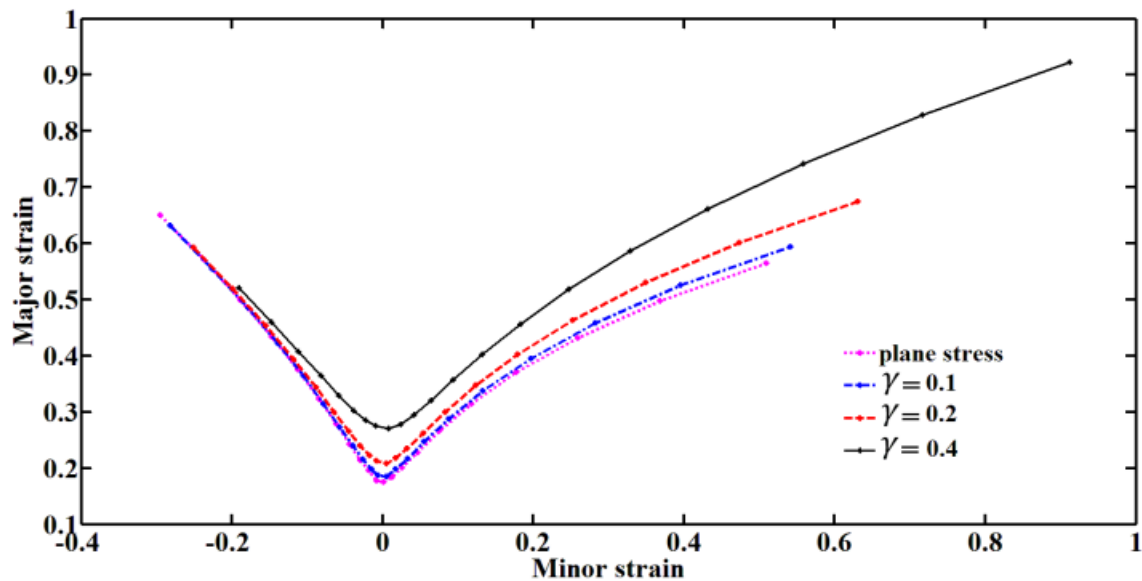


Fig. 2. Forming limit diagrams by considering the through-thickness shear stress

#### 4- Results and Discussion

The M-K criterion code was developed to calculate the theoretical forming limit diagram of AA6016-T4 to consider the through-thickness normal and shear stresses. Fig. 1 shows the influence of different levels of compressive normal stresses on the forming limit diagrams.

As shown in Fig. 1, the formability of the workpiece improves by applying through-thickness normal stress, and the levels of curves are enhanced with an increase in normal stress

In the continuation of this section, the sensitivity of the forming limit curve to the through-thickness shear stress is examined. Fig. 2, indicated the forming limit strains for different amounts of the  $\gamma = \sigma_{32} / \sigma_1$ . According to Fig. 2 by applying the through-thickness shear stress the formability increases and curves move to the right side of the diagram.

#### 5- Conclusion

The most important consequences of this study are below items:

- By extending the M-K theory, the Gotoh plane stress yield function will be able to determine the limit strains in the 3-D stress state.
- The Formability was improved by increasing the through-thickness normal stress and limiting strains increase.
- Similar to the influences of the normal stress on FLD, limit strains shifted upward by applying through-thickness shear stress.

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