



Numerical Investigation of Internal Flow Transition Using Modified γ - Re_{θ} Model

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ABSTRACT: The numerical investigation of Transition is one of the challenging issues in turbulence modeling. In the present study, the coefficients of the γ - Re_{θ} model are modified based on the physics of internal flow transition to capture the entrance length properly. To validate the model, the internal flow is simulated using six test cases. A 3D duct, two smooth axisymmetric pipes, a 3D stenosis pipe, two parallel plates, and the backward-facing step configurations are considered at different Reynolds numbers from 2×10^3 to 3×10^5 . The flow variables, including the average velocity field, friction factor, fully developed friction factor, and the reattachment length are compared against the experimental, theoretical and large eddy simulation results. By comparing the results of average velocity against the semi-empirical relations and experimental data using new coefficients, it is observed the model can estimate the entrance length in accordance with experiments. The earlier coefficients lead to a reduction of entrance length by increasing the Reynolds number. Furthermore, the error percentages reduce by more than 7.6 and 26.7 percent using new coefficients rather than earlier models for fully developed friction factor and reattachment length, respectively.

Review History:

Received: Feb. 22, 2022

Revised: May, 31, 2022

Accepted: Jun. 02, 2022

Available Online: Jun. 09, 2022

Keywords:

Internal flow

γ - Re_{θ} transitional model

Reynolds-averaged

Navier–Stokes equations

Numerical simulation

1- Introduction

The process of changing from laminar to turbulent (turbulentization) flow and vice versa (laminarization) is called transition. There are three different transition mechanisms such as separation-induced transition, natural transition, and bypass transition. The first one occurs due to the exponential growth of Tollmien–Schlichting waves and leads to nonlinear breakage to turbulent [1]. The second is affected by a high turbulence intensity of the free stream and is called bypass transition [2]. The last transition mechanism is separation-induced transition and appears where the laminar boundary layer is separated by the adverse pressure gradient and transition expands inside the separated shear layer [3]. The correlation-based model, γ - Re_{θ} , was developed by Menter et al. [4] to cover the deficiencies of the earlier correlation-based models. The new model uses two transport equations based on local variables (e.g. local pressure gradient, local vorticity, local distance to the wall, and so on) and therefore is compatible with modern Computational Fluid Dynamics (CFD) codes. The model was validated against the basic test cases, such as a two-dimensional turbine blade, and good agreement was achieved against experimental data. Further validation of the model was conducted by Langtry and Menter [5] for a wide variety of test cases such as 2D airfoils, a 3D element flap, a 3D transonic wing, and a full helicopter

configuration. Investigation of transition for external flow received heightened attention and different models including, the e^N method, experimental correlation, and physical-based models developed to predict external transition characteristics. However, minimal attention is given to analyzing internal transition. The lack of an appropriate transition model for internal flows along with necessary experimental correlations leads to modification of the external transition model. Abraham et al. [6] modified two tunable coefficients c_{e2} and $c_{\theta t}$ of the external model based on the fully developed friction factor inside a pipe. c_{e2} and $c_{\theta t}$ are multipliers of E_{γ_2} and $P_{\theta t}$ terms for the external model.

In the present paper in contrast to Abraham, the tunable constants of Menter's model (c_{e2} and $c_{\theta t}$) are modified based on the developing region characteristics such as the entrance length of flow inside a pipe at a variety of Re numbers. To validate the new coefficients and proves they are independent of geometry, the flow inside six different test cases is simulated. Each test case covers one aspect of the transition phenomenon. Therefore, the universality of new coefficients is proved. Some flow variables, including the average velocity field, the Turbulent Kinetic Energy (TKE), the length of the entrance region, the reattachment length, the friction factor, and the fully developed friction factor are investigated at different Reynolds numbers from 2×10^3 to 3×10^5 . The results

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are compared against available data, including experimental, theoretical, and Large Eddy Simulation (LES).

2- Methodology

The flow is incompressible and unsteady. The governing equations of the unsteady flow including the continuity, momentum, transition, and additional equations for turbulence closure are given in Eqs. (1) to (6):

$$\frac{\partial \rho u_i}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[(\mu + \mu_t) \frac{\partial u_i}{\partial x_j} \right] \quad (2)$$

$$\frac{\partial (\rho \gamma)}{\partial t} + \frac{\partial (\rho u_j \gamma)}{\partial x_j} = P_{\gamma 1} - E_{\gamma 1} + P_{\gamma 2} - E_{\gamma 2} + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\gamma} \right) \frac{\partial \gamma}{\partial x_j} \right] \quad (3)$$

$$\frac{\partial (\rho Re_\alpha)}{\partial t} + \frac{\partial (\rho u_j Re_\alpha)}{\partial x_j} = P_\alpha + \frac{\partial}{\partial x_j} \left[\sigma_\alpha (\mu + \mu_t) \frac{\partial Re_\alpha}{\partial x_j} \right] \quad (4)$$

$$\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho u_j k)}{\partial x_j} = \frac{\partial}{\partial x_j} [(\mu + \alpha_k \mu_t) \frac{\partial k}{\partial x_j}] + P_k - D_k \quad (5)$$

$$\frac{\partial (\rho \omega)}{\partial t} + \frac{\partial (\rho u_j \omega)}{\partial x_j} = \frac{\partial}{\partial x_j} [(\mu + \alpha_\omega \mu_t) \frac{\partial \omega}{\partial x_j}] + \alpha \frac{P_k}{\nu_t} - D_\omega + Cd_\omega \quad (6)$$

Six different Test Cases (TC) are considered to evaluate the modified model capabilities including two axisymmetric pipes, a 3D duct, a 3D stenosis pipe, a backward-facing step, and parallel plates.

3- Results and Discussion

An axisymmetric pipe is used to replace the original coefficients with the suitable ones for internal flows. Figs. 1(a) and 1(b) show the variations of centreline velocity in terms of axial location for two different sets of coefficients. The growth of boundary layers leads to an acceleration of the flow core and concurrent with the flow recovery, the value of the centerline velocity reduces and becomes constant in the fully developed region. By increasing Reynolds number, the location of a fully developed region moves downstream due to thinning of boundary layers. However, the original model shows the reverse movement of the fully developed region by increasing the Reynolds number which is in contrast to experiments. Replacing the original coefficients with the new ones modifies the unsuitable behavior of the original models and follows the trend of experiments. The high value of c_{e2} causes an imbalance between transition and destruction sources to prevent the growth of transition sources artificially. Therefore, a lag arises while switching

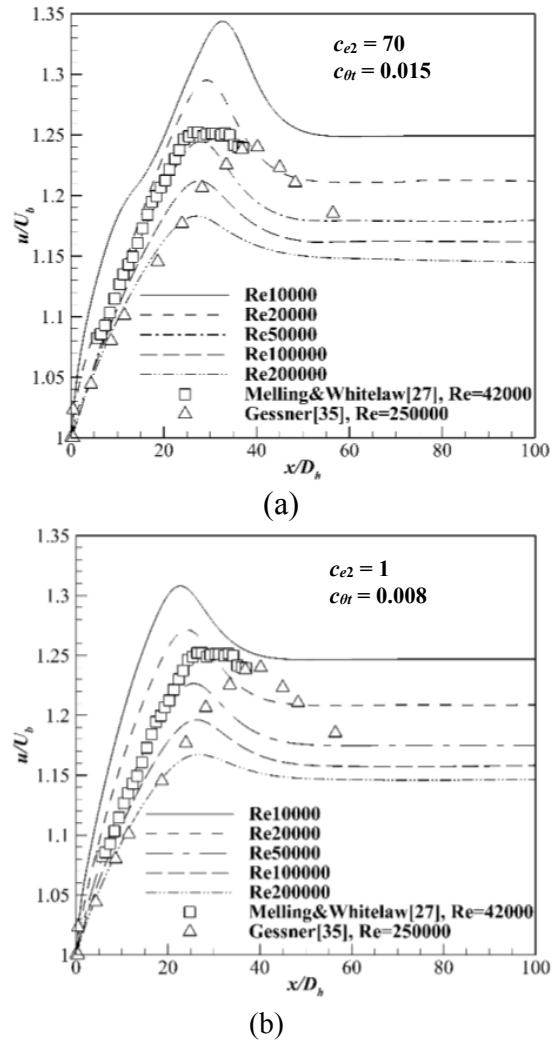


Fig. 1. Centerline pipe velocity profile along the longitudinal axis at different Reynolds numbers for the turbulence intensity ranges from 0.03 to 0.05 (a) Abraham's model (b) New modified model

from one regime to another. The proof of this claim is the disappearance of the imbalance by increasing Reynolds number, due to decay of destruction and growth of transition sources. By increasing Reynolds number, F_{turb} decreases by an exponential rate and overcomes the value of c_{e2} and destruction sources fall-down. The new coefficient creates a suitable balance between transition and destruction sources both at high and low turbulent Reynolds numbers. Therefore, the predicted entrance length using the new modified model is in accordance with the experiments [7]. Promotion of transition source leads to the inclination of flow to bypass transition under the influence of high turbulent Reynolds number, however, the original model shows a delay to bypass transition. The original model predicts natural transition instead of bypass transition at high turbulent Reynolds numbers ($10^4 \leq Re < 2 \times 10^5$) in contrast to experiments [8] and the new modified model. Both unsuitable predictions of

the entrance length and delay to bypass transition are related to the high value of c_{e2} in the original model. At $Re \geq 2 \times 10^5$ along with a range of turbulence intensity (I) from 0.03 to 0.05, laminar and transition regions disappear and the original model operates as a fully turbulent model. However, the fully turbulent flow inside a pipe occurs at $Re \geq 1.3 \times 10^4$ [8], where the slug structures form completely. The difference between the two Reynolds numbers 1.3×10^4 and 2×10^5 is too much and implies the existence of a large lag in the original model.

4- Conclusions

In this paper, the original coefficients were replaced by the new ones, and the procedures of coefficient adjustment were performed according to the physical characteristics of the internal flow such as entrance length. The ability of the new modified transitional SST model was evaluated for six computational domains. The error percentages of the new modified model in all simulations were lower than those of the others.

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HOW TO CITE THIS ARTICLE

M. A. Modaresi, G. Heidarinejad, R. Maddahian, B. Firoozabadi, *Numerical Investigation of Internal Flow Transition Using Modified γ - Re_{θ} Model*, *Amirkabir J. Mech Eng.*, 54(7) (2022) 313-316.

DOI: 10.22060/mej.2022.21138.7383



