



## Large-Amplitude Frequency Analysis of Bi-directional Functionally Graded with Non-Uniform Porous Beams using a Higher Order Shear Deformation Theory

M. Forghani<sup>1</sup>, Y. Bazargan-lari<sup>1\*</sup>, P. Zahedinejad<sup>2</sup>, M. Kazemzadeh Parsi<sup>1</sup>

<sup>1</sup> Department of Mechanical Engineering, Shiraz Branch, Islamic Azad University, Shiraz, Iran

<sup>2</sup> Department of Mechanical Engineering, University of North Texas, North Texas, USA

**ABSTRACT:** This Paper deals with the large amplitude frequency behavior of porous bi-directional functionally graded beams subjected to various boundary conditions which are simply supported, clamped-simply supported, clamped-clamped, and clamped-free utilizing Reddy third-order shear deformation theory and Green's tensor together with the Von Karman geometric nonlinearity. The material properties of the beam change according to power and exponential law in both directions. The equations of motion and associated boundary conditions are derived by means of Hamilton's principle. A generalized differential quadrature method in conjunction with a direct numerical iteration method is selected to solve the system of equations. Demonstrating the convergence of this method, the verification is performed by using extracted results from a previous study based on the Timoshenko beam theory. The results of extensive studies are provided to understand the influences of the different gradient indexes, vibration amplitude ratio, porosity coefficient, Tapered ratio, shear and elastic foundation parameters, and boundary conditions on the Large amplitude vibration frequencies of the bi-directional functionally graded beams. The results reveal that non-linear frequencies increase with the rise of elastic foundation and tapered coefficients and the soar of porosities and material gradients in two directions causes a sharp decrease in non-dimensional frequencies. The results of this study, while carefully examining the frequencies of variable cross-sectional functionally graded beams, are effective in the optimal design of bi-directional beams and are very effective in predicting and detecting failure modes of these beams.

### Review History:

Received: Dec. 23, 2021

Revised: May, 11, 2022

Accepted: Jul. 03, 2022

Available Online: Jul. 15, 2022

### Keywords:

Large-amplitude vibration

Bi-directional functionally graded beam

Porous

Variable cross section

Third order shear deformation theory

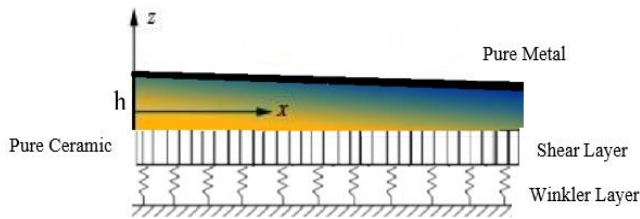
### 1- Introduction

Functionally graded (FG) materials are made of two parts, ceramic and metal. Due to the fact that ceramic structural materials have a low heat transfer coefficient and high-temperature resistance, they can withstand high heat. On the other hand, another material such as metal provides the required flexibility of the structure. It is worth noting that due to continuous changes in mechanical properties, discontinuity problems present in conventional composite structures do not arise in functionally graded materials. Therefore, these materials have been widely used as structural components in modern engineering, nuclear, and aerospace industries, and today there is a wide tendency to investigate these components in various operational and boundary conditions. Some of the research was in accordance with the Euler-Bernoulli theory or classic and were suitable for narrow functionally graded beams. For deeper beams, the classical theory of beams cannot be used due to the reduction of the transverse shear deformation effect and the overestimation of frequencies. In order to overcome the limitations of the classical theory, the beam theory of first-order shear

deformation, known as Timoshenko's theory, has been introduced in relation to the transverse shear deformation of beams. This theory violates the condition of zero shear stress at the upper and lower levels of the beam, so the use of the shear correction factor to calculate the difference between the actual and theoretical stress states is inevitable [1]. In order to more accurately calculate the natural frequencies, displacement, and stress components of medium and thick beams, higher-order shear deformation theories have been presented, in which the use of the shear correction factor is avoided [1]. So far, based on higher-order shear deformation theories, research on the vibration behavior of graded beams has been presented. Aydegdo and Taskin [2] investigated the free vibration behavior of functionally graded beams. The vibration behavior of functionally graded beams in different boundary conditions, using higher-order shear theories, has been presented by Simsek [3]. Then the vibration analysis of graded beams using higher-order shear deformation theories was investigated by Hu Tai [4]. The free vibration behavior of Functionally graded beams under different boundary conditions and based on different shear beam deformation

\*Corresponding author's email: bazarganlari@iaushiraz.ac.ir





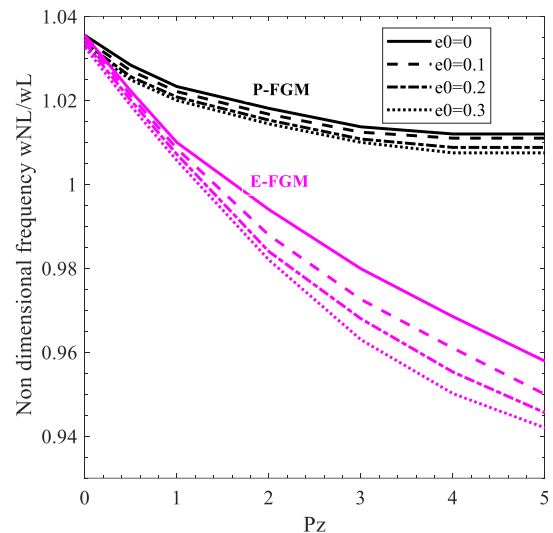
**Fig. 1. Schematic of the variable cross-section of bi-directional FG beam resting on elastic foundation**

Theories have been presented by Chakraborty and Pradhan [5] using the Rayleigh-Ritz method. Most phenomena in the world around us, especially various engineering problems such as vibrations, are inherently nonlinear. Sometimes behaviors are observed in non-linear systems that do not occur in linear systems. Although nonlinear problems can usually be solved by assuming linear behavior around the operating conditions, this is not always justified. The non-linear frequency behavior of Timoshenko beams made of porous materials was presented by Ebrahimi and Zia [6], in which the effect of material distribution, porosity, aspect ratio, and mode number on the vibration behavior of graded beams was carried out. Malekzadeh and Shejaei [7] investigated the surface and non-local effects of non-linear vibrations of non-uniform FG beam by using the theory of Euler Bernoulli's theory and Timoshenko's in connection with Eringen's theory and Hamilton's principle and the method of combined differential quadratic solution and a numerical iterative method was investigated. Forghani et al. [8] investigated the non-linear frequency behavior of functionally graded porous beams with cracks on an elastic substrate using the third-order theory of Reddy.

**2- Methodology**

A bi-directional functionally graded beam with length  $L$  and rectangular cross-section  $b \times h$  on elastic Winkler and shear layers of the bed, consider that  $b$  is the width and  $h$  is the height of the beam. According to Fig. 1, the Cartesian coordinate system  $x, y$  and  $z$  are considered in the direction of length, width, and height of the beam, respectively. The displacement field of the beam is according to the theory of third-order shear deformation and based on some assumptions.

One of the most important higher-order shear deformation theories is Reddy's third-order shear theory. In this theory, unlike classical and first-order shear theories, the cross-sectional area of the beam is no longer considered as a smooth surface. The shear component of the longitudinal displacement increases in such a way as to change the order of the three shear strains in the thickness of the beam so that the shear stress at the upper and lower levels of the beam becomes zero. The equations of motion and the corresponding boundary conditions are converted to algebraic equations by the Differential Quadrature (DQ) method.  $N$  is



**Fig. 2. Variation of nonlinear to linear frequency ratio of Clamped-Clamped (C-C) porous beams versus power law indexes  $p_z, p_x$  for different porosity coefficients ( $l/h=5, \omega_{max} / r = 1$ )**

the number of discretization points and  $A_i, B_i, C_i, D_i$  are the weight constants of the first, second, third, and fourth degrees of differential quadrature respectively, obtained from the Lagrangian interpolation polynomial functions. For discretization, the Gauss-Lobatto-Chebyshev non-uniform distribution method is used.

**3- Results and Discussion**

In order to derive the natural frequencies of the power law and exponentially functionally graded beam on the elastic foundation, first, the convergence behavior of the method is examined and then the comparison with other articles is considered to determine the accuracy of the answers and the accuracy of the results. In Fig. 2 The effect of increasing the porosity coefficients on the non-linear to linear frequency ratio of the bi-directional functionally graded beam has been investigated for two power and exponential models and for different material indices. It is noteworthy that increasing the values of either leads to a gradual decrease in the nonlinear to linear frequency ratio. In this regard, the effect of increasing porosity coefficients in reducing frequency ratios is significant.

**4- Conclusions**

In this paper, the evaluation of the large amplitude frequency of the bi-directional functionally graded porous beams was carried out using the Reddy shear deformation beam theory and Green's tensor in connection with Van Karman's geometric nonlinearity. In this regard, the following results were obtained

- 1) The increase of nonlinear to linear frequency ratio

with respect to amplitude depends on the type of boundary conditions, but this trend is not true for Clamped-Free (C-F) boundary conditions.

2) The effect of increasing the porosity coefficients, power law, and material indices in reducing the non-linear to linear frequency ratio due to the reduction of the stiffness of the bi-directional functionally graded beams.

3) For both Functionally graded beam models, the frequency ratio affected by porosity is more affected by the material gradient index in the longitudinal  $x$  direction than by the material index in the  $z$ -direction.

4) The effect of increasing the section narrowing coefficient on the non-linear frequency behavior of the graded beam.

5) The effect of foundation stiffness coefficients on the nonlinear to linear frequency ratio of an isotropic beam.

### References

- [1] P. Zahedinejad, Free vibration analysis of functionally graded beams resting on elastic foundation in thermal environment, *International Journal of Structural Stability and Dynamics*, 16(07) (2016) 1550029.
- [2] M. Aydogdu, V. Taskin, Free vibration analysis of functionally graded beams with simply supported edges, *Materials & design*, 28(5) (2007) 1651-1656.
- [3] M. Şimşek, Fundamental frequency analysis of functionally graded beams by using different higher-order beam theories, *Nuclear Engineering and Design*, 240(4) (2010) 697-705.
- [4] H.-T. Thai, T.P. Vo, Bending and free vibration of functionally graded beams using various higher-order shear deformation beam theories, *International Journal of Mechanical Sciences*, 62(1) (2012) 57-66.
- [5] S. Chakraverty, K. Pradhan, Free vibration of exponential functionally graded rectangular plates in thermal environment with general boundary conditions, *Aerospace Science and Technology*, 36 (2014) 132-156.
- [6] F. Ebrahimi, M. Zia, Large amplitude nonlinear vibration analysis of functionally graded Timoshenko beams with porosities, *Acta Astronautica*, 116 (2015) 117-125.
- [7] P. Malekzadeh, M. Shojaee, Surface and nonlocal effects on the nonlinear free vibration of non-uniform nanobeams, *Composites Part B: Engineering*, 52 (2013) 84-92.
- [8] M. Forghani, Y. Bazarganlari, P. Zahedinejad, M.J. Kazemzadeh-Parsi, Nonlinear frequency behavior of cracked functionally graded porous beams resting on elastic foundation using Reddy shear deformation theory, *Journal of Vibration and Control*, (2022) 10775463221080213.

#### HOW TO CITE THIS ARTICLE

M. Forghani, Y. Bazargan-lari, P. Zahedinejad, M. Kazemzadeh Parsi, Large-Amplitude Frequency Analysis of Bi-directional Functionally Graded with Non-Uniform Porous Beams using a Higher Order Shear Deformation Theory, *Amirkabir J. Mech. Eng.*, 54(8) (2022) 357-360.

DOI: 10.22060/mej.2022.20916.7336



