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Evaluating the fast method based on proper orthogonal decomposition for radiative heat transfer in a participating medium

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ABSTRACT: The radiative transfer equation models the thermal radiation in a participating medium. Except in specified cases, there is no analytical solution for this equation. Solving the radiative transfer equation with numerical methods is usually time-consuming. This work presents a fast method based on proper orthogonal decomposition to solve the radiative transfer equation. Some variables are selected as independent parameters. The radiative transfer equation for the specified value of these parameters is solved using the discrete ordinates method, and the system responses form the snapshot matrix. The matrix is decomposed singular value decomposition as a product of three matrices. Due to the magnitude of singular values, only a few first columns of these matrices are selected. As a result, the degrees of freedom of the original system are decreased, and a reduced-order model is created. Employing the radial basis functions, the system response, corresponding to any arbitrary input vector (independent parameters), can be approximated with high speed. The results show that the reduced-order method has high accuracy compared to the numerical solution. The complexities of the system do not affect the reduced-order method. Regardless of the characteristics of the medium (the value of independent parameters), the solution time is the order of 0.02 seconds.

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1- Introduction

In high-temperature systems, thermal radiative is the dominant mode of heat transfer. The Radiative Transfer Equation (RTE) is the relationship that can describe the physical behavior of radiative transfer in participating medium [1]. The numerical methods make the RTE a large-scale system. So, finding techniques that increase the computational speed with accuracy is in attendance. Reduced-Order Methods (ROM) project a large-scale system into a smaller one. The Proper Orthogonal Decomposition (POD) is a suitable technique for ROM in many engineering applications [2-4].

From the literature review, no comprehensive research has been done on the use of reduced-order modeling to solve the radiative transfer equation. In some surveys, the POD method had been used to reduce the order of radiative problems. But the RTE was not solved in its general form. Therefore, in this study, a fast method based on POD is introduced to solve the RTE.

2- Problem Formulation

The radiative transfer equation for an absorbing, emitting, and scattering gray medium is written as follows [1],

$$\xi_n \frac{\partial I^n}{\partial x} + \eta_n \frac{\partial I^n}{\partial y} + \beta I^n = \beta S^n; \;\; n = 1, 2, \ldots, m \tag{1}$$

in which I is the radiation intensity, I_b is the black body intensity, κ and σ_s are the absorption and the scattering coefficients respectively, $\beta = \kappa + \sigma_s$ is the extinction coefficient and $\Phi(\mathbf{s}', \mathbf{s})$ is the scattering phase function.

3- Numerical Method

Among the different methods, the discrete ordinates method has many advantages, which has made it popular to solve the RTE [1].

3- 1- Discrete ordinates method

The discrete ordinates method replaces the integrals over solid angle by numerical quadratures. For 2D Cartesian coordinates, and for a direction $\mathbf{s}_n$ with direction cosines ξ_n and η_n Eq. (1) becomes

$$\mathbf{s} \cdot \nabla I(\mathbf{s}) = \kappa I_b - \beta I(\mathbf{s}) + \frac{\sigma_s}{4\pi} \int_{4\pi} I(\mathbf{s}') \Phi(\mathbf{s}', \mathbf{s}) d\Omega' \tag{2}$$

where S^n is the radiative source function and becomes

$$S^n = (1-\omega) I_b + \frac{\omega}{4\pi} \sum_{k=1}^{N} w_k \Phi(\mathbf{s}_n, \mathbf{s}_k) I^k \tag{3}$$

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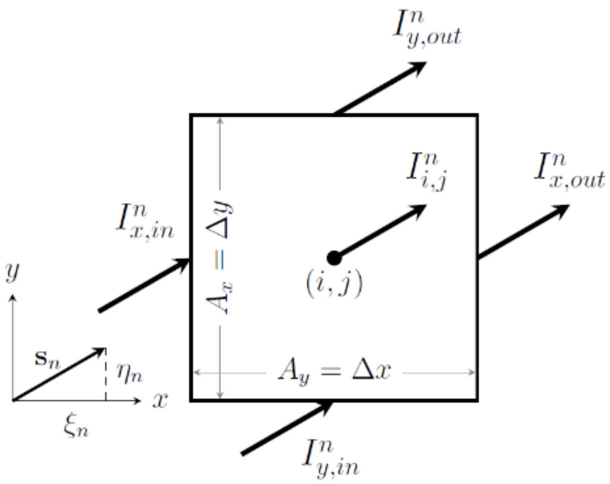

Fig. 1. Radiation intensity in a sample control volume, the discrete ordinates method

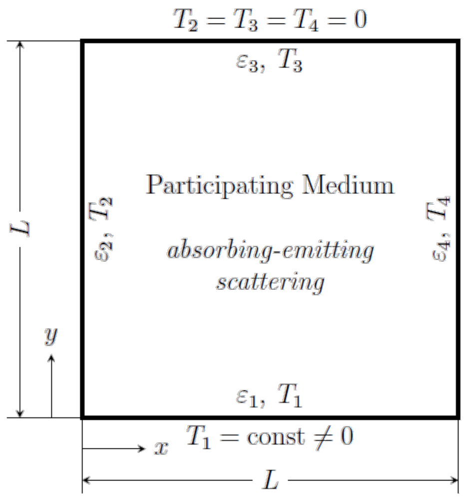

Fig. 2. Schematic of the radiative heat transfer problem

For any discrete ordinate, the volume-averaged intensity of the control volume (Fig. 1) is calculated as follows,

$$I^n_{i,j} = \frac{\beta V S^n_{i,j} + \xi_n A_x I^n_{x,in} / \gamma_x + \eta_n A_y I^n_{y,in} / \gamma_y}{\beta V + \xi_n A_x / \gamma_x + \eta_n A_y / \gamma_y} \tag{4}$$

in which $1/2 \leq \gamma_x, \gamma_y \leq 1$ are weighted differencing.

3- 2- Reduced-order modeling

Reducing the system's Degrees Of Freedom (DOF) is a way to reduce the computation time.

3- 3- Proper orthogonal decomposition

The proper orthogonal decomposition (POD) offers the appropriate bases for the modal analysis of a set of discrete and continuous functions. Assume $F \in \mathbb{R}^{p\times\ell}$ is a matrix and $\{\varphi_i\}_{i=1}^{\ell}$ is a set of orthonormal bases, then F can be expressed as follows,

$$F = \sum_{i=1}^{\ell} \alpha_i \varphi_i = \phi A \tag{5}$$

The Singular Value Decomposition (SVD) calculates the bases satisfying the POD requirement in the sample space. Any matrix has the singular value decomposition.

$$F = \sum_{i=1}^{\ell} \alpha_i \varphi_i = \phi A \tag{6}$$

where $U \in \mathbb{R}^{p\times p}$ and $V \in \mathbb{R}^{\ell\times\ell}$ matrices are orthonormal, and the matrix $\Sigma \in \mathbb{R}^{p\times\ell}$ is a diagonal matrix that its entries are singular values of the F. It can be shown that the matrix U contains the optimal bases for Eq. (5) [5].

3- 4- Radial basis functions

Radial Basis Functions (RBF) are frequently used to approximate a multivariable function by curve fitting through the existing data. The approximation of any function is written as a linear combination of radial functions that can be non-linear.

3- 5- POD-RBF procedure

Using a certain number of input vectors, the snapshot matrix is formed. The proper bases are calculated using the POD. The matrix of amplitudes is as follows,

$$A = \phi^T F \tag{7}$$

The matrix of amplitudes can be written as a linear combination of radial basis functions as below,

$$A = \phi^T F \tag{8}$$

Equation (8) is solved to define matrix B, called radial coefficient matrix. Finally, the F for any desired vector is obtained from the following equation.

$$F(X) \simeq \phi B g(X) \tag{9}$$

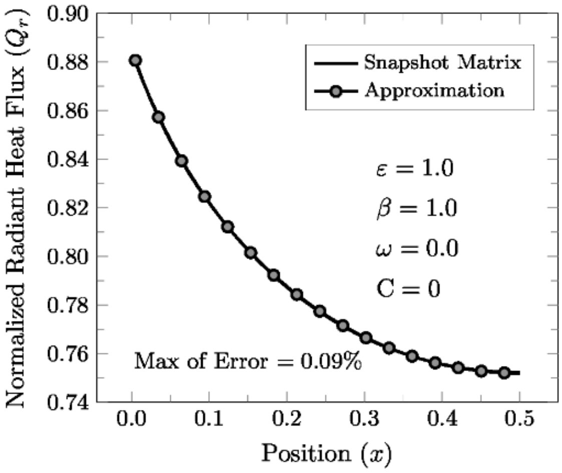

Fig. 3. Comparing the results of POD-RBF approximation with the numerical responses

4- Results and Discussion

A square enclosure, as shown in Fig. 2, is considered. It is assumed that the walls have a constant temperature and are gray and diffuse. The medium is divided into 101 × 101 uniform control volumes. The aim is to investigate the dimensionless radiant flux ($Q_r = q_r / \sigma T_{ref}^4$) on the bottom wall.

Surface emissivities ε, single scattering albedo $\omega = \sigma_s / \beta$, extinction coefficient β, and asymmetry factor C are considered independent parameters. The snapshot matrix is decomposed using the SVD method. So, the orthogonal bases and their corresponding singular values are calculated. The accuracy of the ROM is examined for specific inputs. Fig. 3 illustrates the comparison.

4- 1- The Efficiency of the ROM

The RTE equation is solved, and the efficiency of the combined POD-RBF method is evaluated. For each case, the problem is solved using the DOM and the POD-RBF, and the Central Processing Unit (CPU) time is compared. Table 1 presents the results.

Table 1. CPU time for the numerical solution (DOM) and the Reduced-order (POD-RBF) approximation

Parameters				CPU Time (s)	
ε	β	ω	C	DOM	POD-RBF
0.1	10	0	0	192.23	0.0211
1	1	1	1	2.01	0.0208

5- Conclusions

The POD-RBF model approximates the system response for any arbitrary input vectors. The results show the high accuracy of the presented approach. The CPU time (to evaluate the efficiency of the ROM model) was compared with the DOM results. The results show that the computation time has decreased. On the other hand, the radiation conditions of the medium do not affect the computational cost, and for all modes, it is of the order of 0.02 seconds.

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