



# Nonlocal Vibration of Nanobeam Embedded in Viscoelastic Pasternak Foundation with Longitudinal and Rotational Motions with Surface Effects

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**ABSTRACT:** This paper analyzes the size-dependent vibration of nanoscale beams with simultaneously longitudinal and rotational motions based on nonlocal theory for the optimum design of nanoscale surgical robots. Also, for the first time, a parametric study is performed to explain the surface effects, viscoelastic-Pasternak foundations characteristics, thermal loads, geometric properties, symmetric and asymmetric cross-sections, axial and follower loads on the dynamics and stability of the system. Adopting the Galerkin discretization approach, the reduced-order dynamic model of the system is acquired. Also, analytical and numerical methods are exploited. To ensure the accuracy of the proposed model and method, the present study results are compared and validated with those of published articles. Stability maps and Campbell diagrams are drawn for different working conditions. The results showed that increasing the surface elastic modulus and residual stress improves the vibration frequencies and dynamic instability threshold. It is also found that with increasing system thickness/length, the axial velocity of static instability decreases/increases. In addition, it is observed that the system performance improves with increasing the elastic and shear coefficients of the foundation. The results of the present study significantly help designers and engineers control the vibration of bi-gyroscopic nanoscale robots.

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## 1- Introduction

Bi-gyroscopic structures play substantial roles in diverse engineering fields such as surgical robots, offshore, and electro-mechanics [1, 2]. Due to size reduction in engineering nano-devices, considering the surface effects plays an essential role in the dynamic modeling of nano-systems [3]. It is widely known that by miniaturizing the scale of structures, classical continuum theories cannot correctly estimate the dynamic characteristics of micro/nanoscale systems [4, 5]. The size-dependent vibrations and stability of rotating with axially moving nanobeams with symmetric and asymmetric cross-sections enclosed in a viscoelastic-Pasternak foundation under axial and follower forces by considering surface effects are studied.

## 2- Problem Formulation

A schematic view of a nanobeam simply-supported beam with axial and spinning motion is given in Fig. 1. The beam moves along its axial direction with constant velocity,  $U$ , and spins simultaneously with constant spin velocity,  $\Omega$ . The beam is under an axial force,  $P$ , and distributed tangential force,  $q$ . It is assumed that the system is rested on a viscoelastic-Pasternak foundation with Coefficients of  $k_w$  and  $k_p$ , respectively. Also, the nanobeam is embedded in a viscous medium with a Coefficient of  $c$ . The strain energy of the nanobeam is given by [6]:

$$E_e = -\int_0^L \left( M_z^{\text{local}} \frac{\partial^2 v}{\partial x^2} + M_y^{\text{local}} \frac{\partial^2 w}{\partial x^2} \right) dx \quad (1)$$

where  $M_z^{\text{local}}$ , and  $M_y^{\text{local}}$  are the bending momentum of the nanobeam and defined as follows:

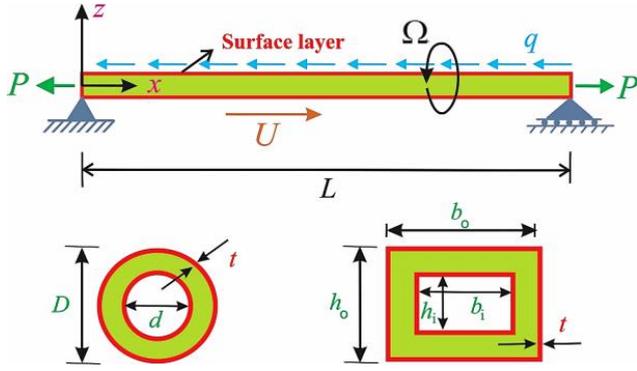
$$\begin{aligned} M_z - (e_0 a)^2 \frac{\partial^2 M_z}{\partial x^2} &= M_z^{\text{local}} \\ M_y - (e_0 a)^2 \frac{\partial^2 M_y}{\partial x^2} &= M_y^{\text{local}} \end{aligned} \quad (2)$$

where  $e_0$  and  $a$  are nonlocal parameters. Also, the kinetic energy of the system can be expressed as:

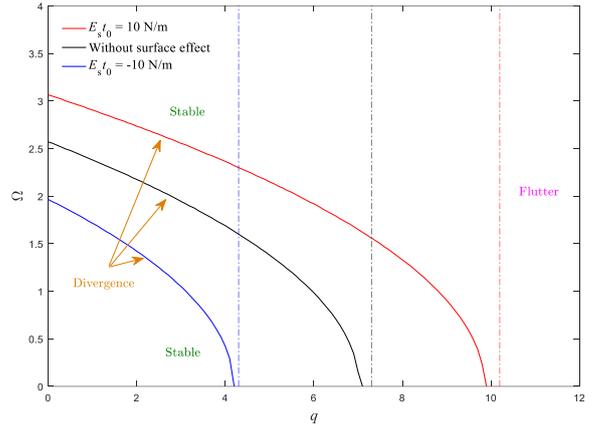
$$\begin{aligned} T_k &= \frac{1}{2} \int_0^L \rho A (\mathbf{V} \cdot \mathbf{V}) dx = \\ & \left( \frac{1}{2} \rho A \int_0^L U^2 + \left( \frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} - \Omega w \right)^2 \right. \\ & \left. + \left( \frac{\partial w}{\partial t} + U \frac{\partial w}{\partial x} + \Omega v \right)^2 \right) dx \end{aligned} \quad (3)$$

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**Fig. 1. Schematic view of a nanobeam with axial and spinning motions under axial and distributed forces**



**Fig. 2. Stability diagram in the q-Ω plane**

Furthermore, the work done by the effects of surface tension can be obtained as:

$$W_s = -\frac{1}{2} \int_0^L H \left[ \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial x} \right)^2 \right] dx \quad (4)$$

in which  $H$  is different for rectangular and circular cross-sections. The work done by the foundation can be obtained as follows:

$$W_{WP} = -\frac{1}{2} \int_0^L (k_w [v^2 + w^2] + k_p \left[ \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial x} \right)^2 \right]) dx \quad (5)$$

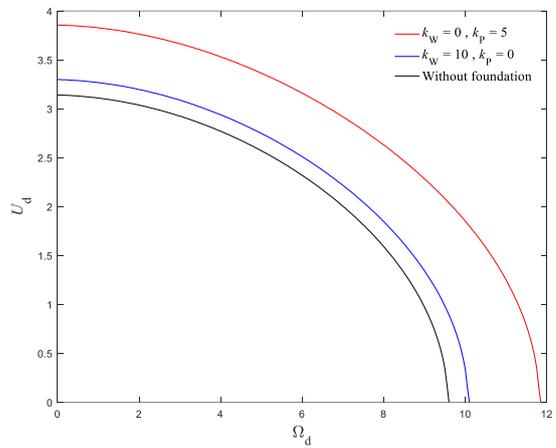
The dynamic governing equations of the system are obtained by exploiting Hamilton's principle. To derive the dimensionless governing equations, the dimensionless parameters are defined, and we introduce two essential parameters of them:

$$\lambda = \frac{I_z}{I_y}, \quad \eta = \frac{e_0 a}{L} \quad (6)$$

in which  $\eta$  is the nonlocal parameter and  $\lambda$  is the inertial ratio in two transverse directions. By adopting the Laplace transform and Galerkin method, discretization of the system equations is given as:

$$\zeta(\chi, t) = \sum_{j=1}^N q_j(t) \varphi_j(\chi) \quad (7)$$

where  $q_j(t)$  is the generalized dimensional coordinate,  $N$  is the number of essential functions,  $\varphi_j$  is the mode for the transverse displacement. The roots of the determinant of the coefficient matrix are system eigenvalues and can be computed in terms of influential factors of the system. The



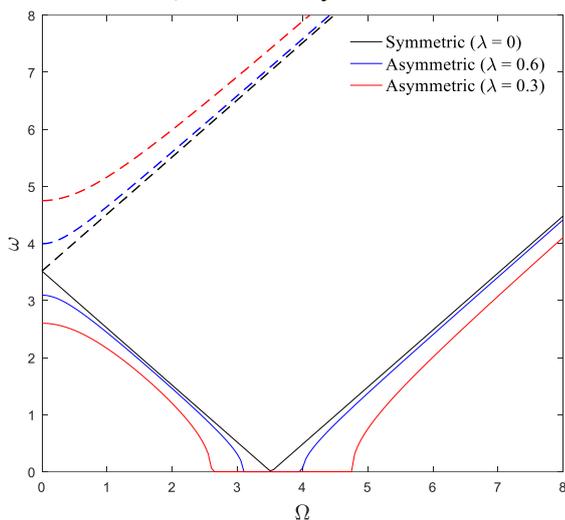
**Fig. 3. Winkler-Pasternak foundation effect on the stability of the system in the  $U_d-\Omega_d$  plane**

imaginary parts of system eigenvalues are considered as frequencies. If a vibration frequency becomes zero, static instability (divergence) happens. In addition, if the imaginary part of the eigenvalue and the system damping is nonzero and positive, respectively, the structure experiences dynamic instability.

### 3- Results and Discussion

Fig. 2 depicts the stability diagram of the system in the  $q-\Omega$  plane. As shown in Fig. 3, surface elastic modulus due to the stiffness-hardening effect can improve the system's stability. Fig. 3 depicts Winkler-Pasternak foundation effects in the  $U_d-\Omega_d$  plane (divergence axial and rotational speeds). According to Fig. 4, the foundation has a practical impact on the system stability, but, compared to Winkler (elastic) foundation, Pasternak (shear) foundation has more impact on the stability of nanobeam due to stiffness-hardening.

Fig. 4 depicts different asymmetric cross-sections of the nanobeam effect on the stability in the Campbell diagram.



**Fig. 4. Asymmetry cross-section and inertial ratio effect**

According to Fig. 4, in asymmetric cross-sections, instead of a border of divergence instability, we have an instability area that, with a decrement of inertia ratio this instability area will increase.

#### 4- Conclusions

For the optimum design of nano surgical robots, a detailed analysis of the dynamical configuration and structural stability of nanobeams with axial and spinning motions subjected to external axial and distributed tangential forces with asymmetric and symmetric rectangular and circular cross-sections is performed. Numerical and analytical procedures are applied to investigate the divergence and flutter instability conditions. It is found that when the system is rested on a foundation, the stiffness-hardening stability of

the system enhances. It is demonstrated that when surface effects are considered, they induce a stabilizing effect on the system. The results showed that the asymmetric cross-section has an area of divergence instability compared to a symmetric cross-section. With a decrease in the inertia ratio, the instability area will increase.

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