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Uncertainty Quantification in the Assessment of the Characteristics of the Electromechanical Impedance Spectrum of a Rectangular Piezoelectric Patch

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ABSTRACT: Electromechanical impedance spectroscopy can be used for damage localization by estimating the electromechanical impedance spectrum with numerical or analytical models. The existence of several sources of uncertainty, however, leads to a significant mismatch between the numerical and experimental results. Therefore, uncertainty quantification for high-frequency coupled electromechanical vibration response of the piezoelectric patch is necessary. Polynomial chaos expansion is an efficient method for assessing uncertainty when dealing with time-consuming models. For the probabilistic analysis of modal features of the impedance spectrum, surrogate models derived by polynomial chaos expansion were used. The statistical moments and probability distributions of the quantity of interest were computed analytically using surrogate models. By post-processing the coefficients of polynomial chaos expansion models with relatively minimal computing cost, global sensitivity analysis was performed to rank the relevance of input variable variation on response variance. According to the results, due to the common uncertainties in the material properties and geometry of the piezoelectric patch, the coefficient of variation in the peak amplitudes is substantially higher than the peak frequencies. In addition, modal frequencies are most sensitive to mechanical properties (compliance and density), whereas modal amplitudes are most sensitive to mechanical damping, electrical permittivity, and the piezoelectric constant.

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1- Introduction

The use of smart materials in various fields of engineering, including structural health monitoring, has grown significantly in recent years. Wafer piezoelectric patches, as a type of intelligent material, have unique characteristics that make them appropriate for real-time health monitoring applications. Electromechanical Impedance Spectroscopy (EMIS) is a vibration-based damage detection method that takes advantage of the simultaneous actuation and sensing capabilities of (PZTs) [1]. Modeling the EMI technique to estimate the PZT response spectrum analytically or numerically is highly beneficial in the damage detection process. However, the presence of various sources of uncertainty regarding material properties and geometry makes experimental validation difficult. Therefore, having knowledge about the impact of the influencing parameters on the EMI in the form of sensitivity indices can guide the model updating process to be handled with the least amount of computational cost.

Polynomial Chaos Expansion (PCE) is one of the most effective techniques for uncertainty quantification by surrogate modeling. The idea is to represent the output response of the original model in a random space spanned by polynomial chaos [2, 3]. The probability distribution of the response [4], statistical moments, and Sobol' sensitivity indices [5] can all be computed by mere post-processing the PCE models.

The present study aims to quantify the uncertainty in the estimation of the piezoelectric patches' EMI spectrum and, in particular, its modal characteristics in high-frequency vibrations. Electrical and mechanical properties, as well as a dimension of a rectangular PZT, are taken as sources of uncertainty in the impedance spectrum estimation. The application of the PCE approach to the probabilistic analysis of the EMI spectrum is one of the study's innovative features. A sensitivity analysis is used to rank the effects of the influencing factors on the modal characteristics of the impedance spectrum.

2- Methodology

For a rectangular patch, the electromechanical (E/M) admittance/impedance can be calculated as follows [6]:

$$Y(\omega) = j\omega\,\bar{C}\left(1-\bar{k}_{31}^{2}\left(1-\frac{1}{\bar{\varphi}\cot(\bar{\varphi})}\right)\right);\; Z(\omega)=Y^{-1}(\omega) \qquad (1)$$

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Table 1. The resonance frequency of a rectangular PZT patch with nominal input parameters

	Resonant frequencies of longitudinal vibration [Hz]			
Experiment	212000	597000	1020000	1496000
Theory	208394	625182	1041971	1458759
Error [%]	1.70	-4.72	-2.15	2.49

$$\bar{k}_{31}^2=\frac{d_{31}^2}{\bar{s}_{11}^E\bar{\varepsilon}_{33}^T},\quad \bar{C}=\bar{\varepsilon}_{33}^T\frac{b_pl_p}{t_p},\quad \bar{\varphi}=\frac{1}{2}\frac{\omega l_a}{\bar{c}},\quad \bar{c}=\sqrt{\frac{1}{\rho\bar{s}_{11}^E}},\qquad (2)$$
$$\bar{s}_{11}^E=s_{11}^E\left(1-j\eta\right),\quad \bar{\varepsilon}_{33}^T=\varepsilon_{11}^E\left(1-j\delta\right)$$

The primary characteristics of the admittance and impedance spectra, which are often used for damage detection, are the modal characteristics. The E/M resonant and anti-resonant frequencies are calculated as the admittance and impedance poles, respectively. Using these spectral characteristics, four Quantities of Interest (QoI) are defined including the mean of the resonant (Y_1) and anti-resonant (Y_2) frequencies, as well as the mean of the peak amplitudes in the real (Y_3) and imaginary (Y_4) parts of the impedance spectrum.

Assuming that QoIs have finite variance, each can be represented by the following expansion:

$$\mathrm{Y}=\mathcal{M}\left(\mathbf{X}\left(\boldsymbol{\Xi}\right)\right)=\sum_{j=0}^{+\infty}\beta_j\Psi_j\left(\boldsymbol{\Xi}\right)\qquad (3)$$

When multivariate polynomials are used as the basis for the expansion, Eq. (3) represents the PCE expansion of the original computational model ($\mathcal{M}$). In practice, the expansion of Eq. (3) is truncated using truncation schemes. Regression analysis allows for the non-intrusive computation of the PCE coefficients [7]. Leave-one-out error is used to assess the accuracy of the PCE model in predicting the original model outcomes [8].

With the PCE surrogate model, the 1st order and total Sobol' indices can be calculated as follows [5]:

$$\hat{S}_i=\sum_{\boldsymbol{\alpha}\in\mathcal{A}_i}a_{\boldsymbol{\alpha}}^2/\sigma_{\hat{Y}}^2\quad \mathcal{A}_i=\left\{\boldsymbol{\alpha}\in\mathcal{A}^{M,p,q}:\alpha_i>0,\alpha_{j\neq i}=0\right\}\qquad (4)$$

$$\hat{S}_i^{\mathrm{T}}=\sum_{\boldsymbol{\alpha}\in\mathcal{A}_i^T}a_{\boldsymbol{\alpha}}^2/\sigma_{\hat{Y}}^2\quad \mathcal{A}_i^T=\left\{\boldsymbol{\alpha}\in\mathcal{A}^{M,p,q}:\alpha_i>0\right\}\qquad (5)$$

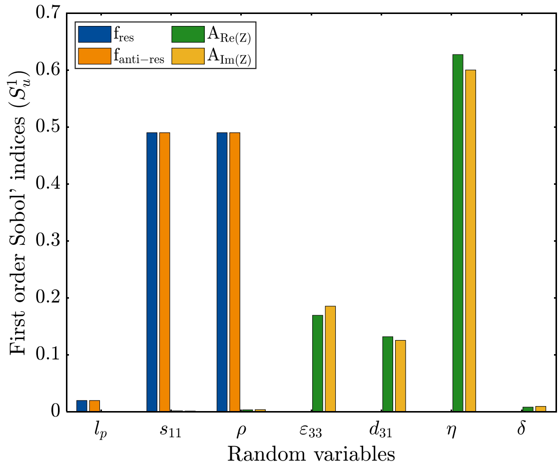

Fig. 1. First order Sobol's sensitivity indices for Random Variables (RVs)

3- Results and Discussion

The EMI spectra for a rectangular PZT are first estimated using the nominal material properties and dimensions. Table 1 compares the model's E/M resonant frequencies for longitudinal vibrations with the experimental data from reference [6].

In the next step, the EMI spectral characteristics are investigated probabilistically using PCE. The seven random variables, which include patch length, longitudinal compliance, density, permittivity, piezoelectric constant, and mechanical and electrical dissipation coefficients, are assumed to have a uniform distribution with mean values and coefficients of variation supplied by the manufacturer. For Sobol's sensitivity indices and QoIs probability distribution, the PCE models surrogate the original computational models to lower the computational costs compared to the Monte Carlo method (12870 calls to the original model relative to 90 calls by the Monte Carlo method).

Figs. 1 and 2 show the first order and total Sobol' indices for the QoIs. The findings show that the resonant and anti-resonant frequencies are particularly sensitive to the mechanical properties of the piezoelectric patch, including its compliance and density. On the other hand, there is almost no correlation between these QoIs and the piezoelectric patch's electrical, E/M, and damping properties. In terms of the amplitude of the EMI spectrum, mechanical dissipation, electrical permittivity, and piezoelectric constant are the most influential factors.

4- Conclusions

A probabilistic analysis of the vibration response of a rectangular piezoelectric patch in terms of EMI was performed. The PCE was used to surrogate the original model and quantify uncertainty. According to the results,

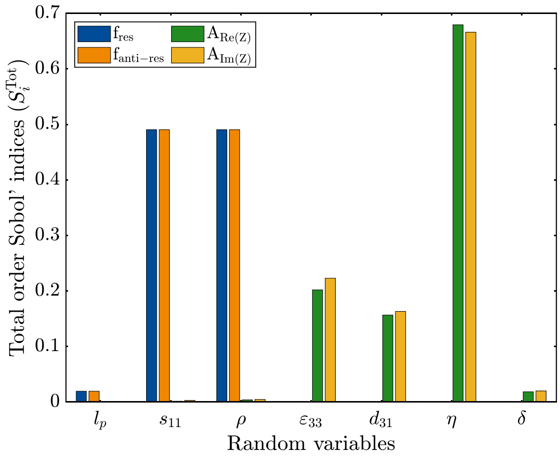

Fig. 2. Total Sobol's sensitivity indices for RVs

for a common coefficient of variation for the input random variables, the coefficient of variation of the amplitude of the impedance spectrum is much larger than that of the modal frequencies. Modal frequencies are most significantly influenced by the mechanical properties of the patch (compliance and density). Meanwhile, the modal amplitudes showed the most sensitivity to the piezoelectric constant, electrical permittivity coefficient, and mechanical dissipation coefficient. The modal quantities of the impedance spectrum are also barely impacted by the RVs' interaction effects.

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