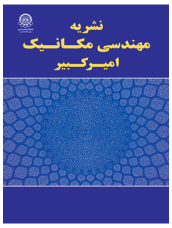

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Stabilization of Reduced Order Model for Convection-Diffusion Problems Based on Dynamic Mode Decomposition at High Reynolds Numbers Using Eddy Viscosity Approach

M. K. Moayyedi[1,2]*, F. Bigdeloo[2], F. Sabaghzadeghan[1]

[1] CFD, Turbulence, and Combustion Research Lab., Department of Mechanical Engineering, University of Qom, Qom, Iran
[2] Space Science and Earth Atmosphere Research Lab., Department of Mechanical Engineering, University of Qom, Qom, Iran

ABSTRACT: Since the analytical methods have a low accuracy and numerical algorithms are time-consuming with hardware limitations, therefore researchers are interested to develop models with high speed and efficiency. The reduced order model is the method that could be an alternative approach for simulating dynamical systems. These models are mainly developed based on the calculation of the dynamical systems' effective structures. The dynamic mode decomposition method is one of the methods for calculating these basic structures. In this study, using this model and based on the principles of dynamical systems, a reduced order model has been developed for the Burgers equation. The results show that if the Reynolds number increases then the effects of the viscous term in the governing equation are decreased, accordingly the required dissipation of the system to stabilize the numerical solution is reduced. Also, due to the incompleteness of the modes which are selected in the order reduction procedure, the dissipation level of the surrogate model is reduced more. Therefore, by creating an artificial dissipation called the eddy viscosity approach, the stability of the model is enhanced. Finally, by comparing the results obtained from the reduced order model and direct numerical simulation, the accuracy of this model is proven.

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1- Introduction

A newer approach to extracting the basic structures of a dynamic system is the dynamic mode decomposition method introduced by Schmid [1]. Dynamic Mode Decomposition (DMD) is a post-processing method that is extracted from the original data information related to the dynamical system. Today, researchers have turned to this method to simulate turbulent flows and nonlinear equations. For example, Rowley et al. [2] used the dynamic mode decomposition method to simulate the flow of a large-scale jet. Hu et al. [3] also investigated the flow of a centrifugal compressor using the dynamic mode decomposition method. Duke et al. [4] investigated the growth rate of flow instability using the dynamic mode decomposition method.

2- Governing Equations

Berger's equation is a differential equation obtained by simplifying the Navier-Stokes equations assuming the absence of pressure changes, and the nonlinear term is the basis to address the turbulent behaviors of flow like the Navier-Stokes equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial x^2} \tag{1}$$

Where *Re* is Reynolds number. A numerical simulation of the Burgers equation has been made by using the first-order upwind method for the nonlinear term and the second-order central difference method for the diffusion term. Also, time integration is performed using the fourth order Runge - Kutta Scheme.

2- 1- Reduced order model based on dynamic mode decomposition

Reduced order modeling is a technique that can reduce computational complexity or computer storage requirements. This can simplify the analysis, control, and design with alternative Reduced Order Models (ROMs). Finally, the equation of the reduced order model will be obtained as a first-order ordinary differential equation for the time-dependent modal coefficients:

$$\frac{da^k(t)}{dt} + \tilde{A}_{kij} \times a^i(t) + \tilde{B}_{ki} \times a^i(t) + \tilde{C}_k = 0 \tag{2}$$

2- 2- Stabilization of reduced order model using eddy viscosity approach

The system of ordinary differential equations obtained from

*Corresponding author's email: moayyedi@qom.ac.ir

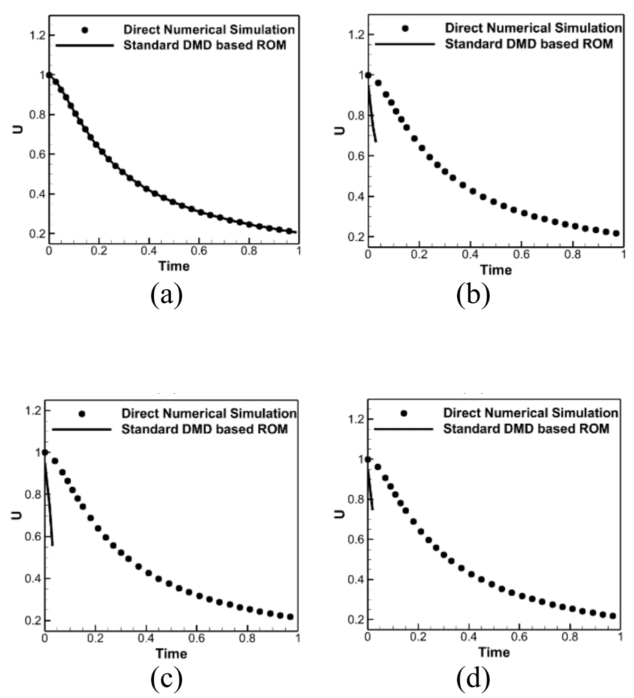

Fig. 1. Comparison between the Prediction of Standard Reduced Order Model and Direct Numerical Simulation of Burgers Equation on x=0.25 at Reynolds Numbers of (a) 100, (b) 1000, (c) 2000, and (d) 5000

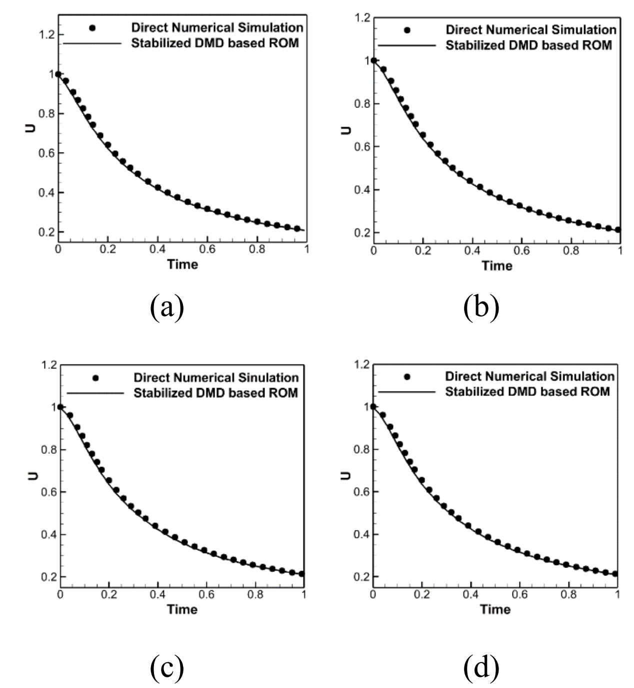

Fig. 2. Comparison between the Prediction of Stabilized Reduced Order Model and Direct Numerical Simulation of Burgers Equation on x=0.25 at Reynolds Numbers of (a) 100, (b) 1000, (c) 2000, and (d) 5000

the Galerkin projection to model the dynamics of the system may be unstable. In this study, as the Reynolds number increases, the viscous term in the Burgers equation will have less effect. Also, neglecting the effects of some modes in the final form of the reduced order model is an important factor in reducing the required dissipation and the stability of the model responses. To correct these effects and compensate for the lost dissipation, an artificial eddy viscosity term is added to the model as a linear and constant term to guarantee the system stability:

$$\begin{aligned} B_k^2 &= \langle \nu_e \nabla^2 \overline{u}, \phi_k \rangle \\ B_{ki}^1 &= \langle \nu_e \nabla^2 \phi_i ; \phi_k \rangle \end{aligned} \tag{3}$$

3- Results and Discussion

In this study, the direct numerical simulation method was used to numerically solve Berger's equation, and the time step was assumed to be 0.001. It should be noted that the numerical solution of this equation has been done for one unit of non-dimensional time and for Reynolds numbers of 100, 1000, 2000, and 5000. To investigate the time-dependent behavior of the DMD-based ROM in predicting the response of Berger's equation, the results of the reduced order model and the direct numerical solution method were calculated at a specific location point ($X = 0.25$) and its data are given in Fig. 1 for different Reynolds numbers. As it is clear in the figure, for Reynolds numbers greater than 1000, the results of the reduced order model diverge, and meaningless values are obtained that cannot be shown in the diagram. A comparison is made in Fig. 2 between the results of direct numerical simulation and stabilized reduced order model. So, by using the method of stabilization of the reduced order model, the accuracy of the results obtained from the model at different times has been seen for all Reynolds numbers.

4- Conclusion

In order to develop a physics-Informed reduced order model, the projection of the governing equation in the modal space has been used. This model at low Reynolds numbers has a good accuracy, which is due to the dominance of the diffusion term in the equation and the effect of increasing its dissipation effect. In this situation, the results of the standard reduced order model based on the dynamic mode decomposition method have good accuracy with the related data obtained from the direct numerical simulation. In the development of the reduced order model, the approach of order reduction is based on removing the effect of some modes. This issue is also an important factor in reducing the required dissipation and the stability of the responses of the reduced order model.

By increasing the Reynolds number to higher values, such as 1000, 2000, and 5000 in the present study, the required dissipation of the dynamical system is reduced similar to a turbulent flow. In order to stabilize and compensate for this lost dissipation, an artificial viscosity approach called eddy viscosity based on turbulent flow modeling concepts will be used as a linear and constant term in the ROM equation. These expressions are used as a substitute for the effect of modes that are removed in the order reduction procedure and are similar to the turbulent flow simulation approach for modeling small-scale structures that have a lower energy level. Therefore, the reduced order model reaches a stable form and will give acceptable results in different Reynolds numbers and in all time steps.

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