Design and Implementation of Sliding Mode Control with the Modified Force for an Inverted Pendulum with Nonlinear Friction

F. Shokouhi, B. Hossein-khani, A. H. Davaie Markazi*

Department of Mechanical Engineering, Iran University of Science and Technology (IUST), Tehran, Iran

**ABSTRACT:** The purpose of this study is to present a method for controlling an inverted pendulum in the presence of nonlinear and indeterminate friction force between the moving cart and its straight guide rail. Control of an Inverted pendulum, as an Under-actuated Mechanical System, is facing challenges from theoretical and experimental aspects. To deal with such challenges, a new method is proposed in this paper. The method is based on an approximate input-output linearization of the inverted pendulum dynamic model for which a modified sliding mode control is proposed. For experimental determination of the bound of friction force, an inverted pendulum with a moving cart is designed and built. The moving cart and its rail are intentionally designed and built such that the resulting friction force is nonlinear, uncertain, and state-varying. The upper bound of the friction force is obtained experimentally and its average value is added to the control input obtained from the conventional sliding mode controller. Experimental verifications depict the success of the proposed control method in preserving the closed-loop stability under the challenging case of dealing with a large nonlinear friction force.

**Keywords:** Sliding mode control, Inverted pendulum on cart, Constant rate reaching law, Nonlinear unknown variable friction

1- Introduction

Control of Under-actuated Mechanical Systems (UMSs) has been studied for many decades and is still one of the most active research areas due to the theoretical and experimental challenges involved [1]. The Inverted Pendulum on Cart (IPOC) system is one of the most common benchmark problems for new control algorithms.

Complex nonlinearities (such as friction, saturation, backlash, or hysteresis) make the linearization process of systems complicated and sometimes impossible. In particular, ignored friction may cause limit cycles, stick-slip, or significant steady-state tracking errors in the system response [2, 3]. One of the apparent properties of robust controllers is controlling a system in the presence of uncertainties and bounded unknown external disturbances in the natural environment. The presence of nonlinear unknown variable friction is one of these examples. This study proposes a robust controller based on the Sliding Mode Control (SMC) method with modified control force instead of finding a precise model of nonlinear unknown variable friction.

In addition to previous references, there are several articles on inverted pendulum control with nonlinear friction. These papers can be divided into two general categories: In the first group, only the pendulum rotation mode control was considered to avoid the UMS challenge, and parameters of the friction model were obtained experimentally. In most such studies, the magnitude of the friction is considered small; for example, in Ref. [4]. In a few articles, the under-actuated model has been considered, and the zero dynamic control method of the system has been used.

2- Dynamic Frictionless Model

As shown in Fig. 1, the inverted pendulum is connected to the moving cart using a rotating shaft. The objective is to retain the pendulum near the upright (unstable) equilibrium point by applying force to the moving cart.

2- 1- Approximate input-output feedback linearization (IOFL)

For the SMC method to be applicable to IPOC, it must be converted to a standard form by the Input-Output Feedback Linearization (IOFL) technique. For this purpose, a suitable diffeomorphism, namely a transformation matrix including the output and its derivatives [5].

3- Laboratory Implementation

Using an inverted pendulum hinged on the moving cart, the laboratory system implements the control algorithm. The Moving cart can move on a straight rail with a length of 2 [m]. A belt drives the moving cart of the pendulum, which...
by applying force near the upright (unstable) equilibrium point connected to the moving cart using a rotating model were obtained experimentally. In most challenge, and parameters of the friction IPOC, it must be converted to a standard form feedback linearization (IOFL) by the Input-Output Feedback Linearization.

### 3. Laboratory Implementation

Moving cart can move on a straight rail with the moving cart moves is deliberately designed to bring high and variable friction to the base (Fig. 2).

**Fig. 1. The structure of the inverted pendulum and the moving cart system**

**Fig. 2. Inverted pendulum made with high friction between the moving cart and the rail.**

belt is moved by the output shaft of the gearbox connected to the direct-current motor. The rail on which the moving cart moves is deliberately designed to bring high and variable friction to the base (Fig. 2).

### 4- Comparison of Simulation and Experimental Results

In the continuation of the paper, the control behavior of the loop system is investigated depending on both theoretical and experimental methods to reveal the adverse effects of nonlinear unknown variable friction in the control process. Despite the lack of an accurate mathematical model for friction and just using its average amount, the proposed force-modified SMC controller managed to maintain the position of the cart and pendulum around the equilibrium points successfully.

The graphs in Fig. 3 and Fig. 4 show the theoretical and experimental positions of the moving cart and the angular position of the pendulum, respectively. Taking into account the experimental data of friction force in the dynamic model of the system and then using the SMC method with modified force, the solid line for the theoretical state is obtained. The dotted line shows the experimental results of the real system controlled by the SMC method with control force modification.

Considering the experimental friction force in the dynamic model of the system, Fig. 5 shows the amount of force applied by the controller to the moving cart for the theoretical case. The dotted trajectory shows the experimental results of the SMC method with a modified control force.

### 5- Conclusions

An attempt was made in this study to investigate how to deal with high and nonlinear frictions in an inverted pendulum. The main goal of this study was to maintain the angle of an inverted pendulum at the upper unstable equilibrium point and the position of its movable base around the origin in the presence of nonlinear unknown variable friction. An attempt was made to obtain a lower and upper limit for the unknown variable nonlinear friction between the moving base and the rail by experimental methods. Its near-average value, called the corrected force was then added to the control force obtained by the SMC method to bring it into the swinging base of the pendulum. As a first step, the closed-loop behavior of the system was simulated using the...
output of the SMC controller. Despite such adverse effects, the pendulum angle and the position of the moving base position are retained within acceptable bounds.

References