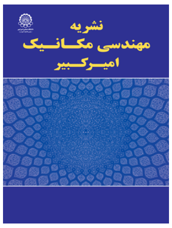

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State-space approach for bending analysis of functionally graded piezoelectric plate using five-variable refined plate theory

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ABSTRACT: In this paper, an analytical solution for bending analysis of functionally-graded piezoelectric plate with two simply-supported parallel edges and two other arbitrary boundary conditions under uniformly-distributed transverse loading is presented. The five-variable refined plate theory is employed for describing the displacement field. This theory, despite the few numbers of unknown variables, predicts a parabolic distribution for transverse shear stresses across the thickness and also considers the thickness-stretching effect. The governing equations are obtained using Hamilton's principle and Maxwell's equation. The Levy-type solution in conjunction with the state-space approach is used to solve them. Comparing the results with those obtained by the higher-order shear theories and Abaqus finite element simulation confirms the accuracy and efficiency of the proposed method. It can be seen that for the length-to-thickness ratio of 10 and the power-law index of 0.5, the value of non-dimensional deflection of the plate with the clamped boundary condition is 0.3327, which has the largest amount of stiffness, while the value of the non-dimensional deflection of the plate with two parallel free boundary condition edges having the lowest amount of stiffness is 2.2036. In addition, for the plate with a clamped boundary condition and length-to-thickness ratio of 10, with the increase of the power index from 0.5 to 10, the value of displacement changes from 0.3327 to 0.3545, which means an increase of about 6%.

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1- Introduction

Functionally-graded (FG) materials could be considered as special composites which their properties change gradually from one surface to another. Four-variable refined plate theory with fewer unknowns compared with the conventional high-order shear deformation theories satisfies the free surfaces' zero traction condition, predicts a parabolic variation of transverse shear stresses through the plate thickness and provides very good results for the analysis of thin and thick plates. Thai and Choi [1] presented a five-variable quasi-3D refined theory by considering the effect of stretching along the thickness.

Piezoelectric materials as a subset of smart materials are extensively used as actuators and sensors in different engineering and industrial applications owing to their electromechanical coupling properties, high accuracy, wide bandwidth, and quick response.

The state-space approach is an effective analytical method for solving ordinary differential equations. This process reduces the higher-order differential equations to a system of first-order differential equations. The first-order differential equations can be easily solved by matrix methods in terms of eigenvalues and eigenvectors.

The main innovation of this research is the use of the five-variable quasi-three-dimensional refined theory to express the displacement field of the functionally graded piezoelectric plate which leads to the consideration of normal strain along the plate thickness. Using Levy's method in conjunction with the state-space approach to analytically solve the governing equations of the plate with different boundary conditions is another aspect of the innovation of this research. In addition, in this investigation, Maxwell's equation coupled with the governing mechanical equations has been used to calculate the electric potential created in the plate.

2- Methodology

Figure 1 shows a schematic view of a rectangular FG piezoelectric plate. It is assumed that the plate has two opposite simply supported edges at x=0 and x=a while two other edges have arbitrary boundary conditions. The distributed uniform mechanical load $q(x,y)$ is applied to the top face of the structure.

The displacement field is defined using the assumptions of the five-variable refined theory [1]:

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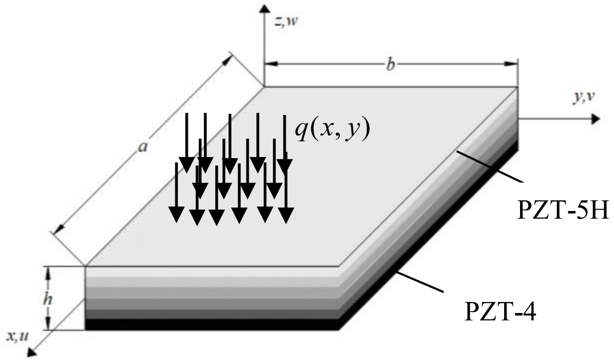

Fig. 1. Geometry of functionally graded piezoelectric plate

$$u(x,y,z)=u_0(x,y)-z\frac{\partial w_b(x,y)}{\partial x}-f(z)\frac{\partial w_s(x,y)}{\partial x}$$
$$v(x,y,z)=v_0(x,y)-z\frac{\partial w_b(x,y)}{\partial y}-f(z)\frac{\partial w_s(x,y)}{\partial y} \quad (1)$$
$$w(x,y,z)=w_b(x,y)+w_s(x,y)+g(z)\ w_z(x,y)$$

The coupled constitutive equations for the piezoelectric layer are expressed as [2]:

$$\sigma=c\varepsilon-e\mathrm{E}$$
$$D=e^T\varepsilon+\eta\mathrm{E} \quad (2)$$

Considering Levy's solution procedure, the components of displacement and electrostatic potential satisfying simply supported conditions at x=0 and x=a are expressed as follows [3]:

$$u_0(x,y)=\sum_{m=1}^{\infty}U_m(y)\cos\alpha x,$$
$$v_0(x,y)=\sum_{m=1}^{\infty}V_m(y)\sin\alpha x,$$
$$w_b(x,y)=\sum_{m=1}^{\infty}W_{bm}(y)\sin\alpha x,$$
$$w_s(x,y)=\sum_{m=1}^{\infty}W_{sm}(y)\sin\alpha x, \quad (3)$$
$$w_z(x,y)=\sum_{m=1}^{\infty}W_{zm}(y)\sin\alpha x,$$
$$\varphi(x,y)=\sum_{m=1}^{\infty}\varphi_m(y)\sin\alpha x$$

Table 1. Comparison of the non-dimensional deflection of the FG Al/Al_2O_3 for different power law indices (a/h=10)

Power index	0	2	8	10
Present	0.4664	1.1940	1.5335	1.5872
[4]	0.4664	1.1939	1.5344	1.5874

By substituting the above expressions in governing equations, six coupled fourth-order differential equations are obtained. These equations can be solved using the state-space approach [3].

3- Discussion and Results

To verify the accuracy and efficiency of the current method, in Table 1 the non-dimensional deflection of the FG Al/Al2O3 is compared with the results reported in [4]. As observed, the theory and method utilized in this study lead to quite accurate results. It can also be seen, by increasing the power-law index, the displacement values increase.

The effect of the power-law index on the distribution of non-dimensional deflection across the width of the SCSC smart FG plate is illustrated in Figure 2. As seen, increasing the power-law index causes the transverse deflection to increase because the elastic modulus of PZT-5H is lower than PZT-4. This figure also reveals that the deflection value is zero at y=0 and y=b according to the clamped boundary conditions at these two edges. At the same time, as expected, the curves possess a maximum value in the middle of the plate width (y=b/2).

Figure 3 represents the effect of boundary conditions on the variations of non-dimensional transverse shear stress across the thickness of the FG piezoelectric plate. The transverse shear stress has a parabolic distribution through the plate thickness for all boundary conditions, and its maximum value occurs at the middle plane, resulting from the utilized plate theory. Additionally, plates with SCSC and SFSF boundary conditions have the lowest and highest value for deflection, respectively due to their highest and lowest stiffness values.

4- Conclusions

The main finding of this research can be listed as:

The five-variable refined plate theory shows very good accuracy compared to other high-order shear theories and the three-dimensional solution of Abaqus, due to the consideration of stretching effect along the thickness and non-zero normal strain in the z-direction.

Levy's solution and state space approach are very efficient and accurate analytical tools for solving the governing equations of plates. The state space method makes the solution very easy by converting higher-order ordinary differential equations into a system of first-order differential equations.

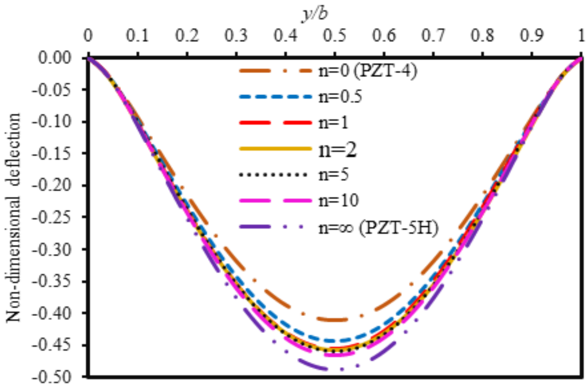

Fig. 2. Non-dimensional deflection of smart FG plate considering different power-law indices (BCs.: SCSC, a/h=5).

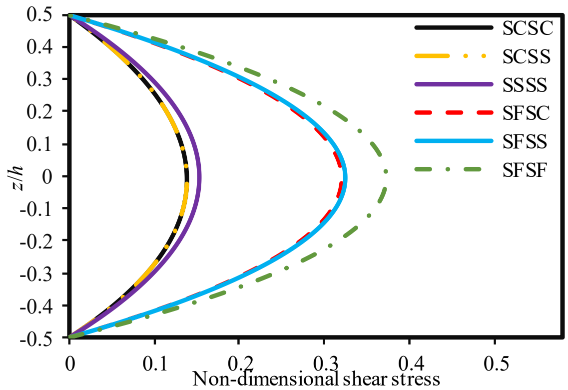

Fig. 3. Variation of non-dimensional transverse shear stress through the smart FG plate thickness (n=2, a/h=5).

By increasing the length-to-thickness ratio and consequently decreasing the amount of plate stiffness, the corresponding deflection, normal stress, and shear stress values will increase.

Having more constraints at the plate boundaries increases the structural stiffness resulting in a reduction in deflection, normal stress, and shear stress values.

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