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Buckling analysis of tapered laminated composite channel-section beam-columns subjected to combined axial load and end moment

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ABSTRACT: Nowadays, the practical applications of shell elements such as beams having thin-wall cross-sections are increasing greatly in various fields of engineering including aerospace, nuclear, marine, and automotive industries. This is due to their ability to optimally use structural materials and simultaneously reduce the total weight of the structure. Fiber polymer composites also have different conspicuous properties such as high stiffness-to-weight and strength-to-weight ratios, corrosion resistance, and high strength. Therefore, laminated composite C-section beam elements simultaneously possess both the beneficial features of fiber-reinforced composite materials and thin-walled cross-sections at the same time. Motivated by these facts, in this research, the flexural-torsional stability of multi-layer fibrous composite tapered beam-columns with channel-section subjected to axial and bending loads is investigated. For this purpose, the total potential energy governing the problem is extracted based on Vlasov's model for small non-uniform torsion along with the classical laminated plate theory. Then, using Ritz's methodology as an analytical solution technique, the endurable buckling load is calculated. Eventually, the effect of important parameters such as stacking sequences, fiber composite materials, boundary conditions, axial load eccentricity, and axial preloading on the linear buckling capacity of double-tapered multi-layer composite beam-column with channel-section under axial load and end moment is investigated.

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1- Introduction

Due to the importance of using thin-walled laminated fibrous composite structural components having constant and/or variable cross-sections in different engineering fields such as axles of vehicles, helicopter rotors, wind turbine blades, and especially aircraft wings, the static and dynamic analyses of thin-walled structural elements with various end conditions under different loading cases have been widely studied in recent decades [1-4]. Based on these facts, in the current study, the overall flexural-torsional buckling response of tapered composite C-shaped beam-column exposed to axial-transverse loadings is investigated using the Ritz's method in the framework of the Classical Laminated Plate Theory (CLPT) and Vlasov's model for non-uniform torsion.

2- The Variational Formulation

A schematic representation of a tapered laminated composite C-shaped beam-column with length L subjected to transverse and axial loadings is shown in Fig. 1. The orthogonal right-hand Cartesian coordinate system (x, y, z) is adopted, wherein x denotes the longitudinal axis and y and z are the first and second principal bending axes parallel to the flanges and web, respectively. The origin of these axes (O) is

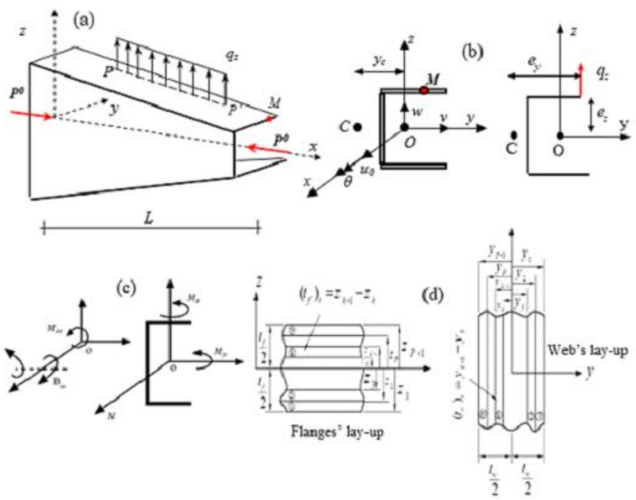

Fig. 1. (a) Schematic representation of axially/transversely loaded thin-walled beam with varying C-shaped cross-section, (b) Displacement fields and load eccentricity parameter, (c) The stress resultant parameters, (d) Laminate configurations.

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located at the centroid of the cross-section. The shear point C is known by its coordinates (yc) in the reference fixed in centroid O.

Based on the small displacements assumption and Vlasov's thin-walled beam theory for non-uniform torsion, the displacement fields can be expressed as [5]:

$$U_{(x,y,z)} = u_{0(x)} - y\frac{\partial v_{(x)}}{\partial x} - z\frac{\partial w_{(x)}}{\partial x} - \phi_{(y,z)}\frac{\partial \theta_{(x)}}{\partial x} \tag{1}$$

$$V_{(x,y,z)} = v_{(x)} - z\theta_{(x)} \tag{2}$$

$$W_{(x,y,z)} = w_{(x)} + (y - y_{c(x)})\theta_{(x)} \tag{3}$$

where U, V, W stand for to the axial, lateral and vertical displacement components along the x, y, z direction, respectively, whereas u, v, w are the kinematic quantities defined at the reference surface, the term $\phi(y,z)$ refers to the warping function, and θ is the twisting angle. In this research, the variational formulation governing the flexural-torsional buckling is extracted on the basis of the stationary state as what follows [4]:

$$\delta\Pi = \delta U_l + \delta U_0 - \delta W_e = 0 \tag{4}$$

In this formulation, δ denotes a variational operator. U_l and U_0 represent the elastic strain energy and the strain energy due to the effects of the initial stresses, respectively. W_e denotes work done by externally applied loads. The expression of the first variation of total potential energy is obtained as

$$\begin{aligned}\delta\Pi &= \int_L \begin{pmatrix} (EA)_{com}u_0'\delta u_0' + (EI_z)_{com}v''\delta v'' \\ +(EI_y)_{com}w''\delta w'' \\ +(EI_\phi)_{com}\theta''\delta\theta'' + (GJ)_{com}\theta'\delta\theta' \end{pmatrix} dx \\ &+ \int_L \begin{pmatrix} P^0 \begin{pmatrix} v'\delta v' + w'\delta w' + y_c'^2\theta\delta\theta \\ -\left(y_c y_c'\right)'\theta\delta\theta + y_c w''\delta\theta + y_c\theta\delta w'' \\ +\left(\dfrac{I_z + I_y}{A} + y_c^2\right)\theta'\delta\theta' \end{pmatrix} \\ -\left(M_y^* v''\delta\theta + M_y^*\theta\delta v''\right) \end{pmatrix} dx \\ &- \int_L \left(q_z\delta w + q_z e_y\delta\theta - q_z e_z\theta\delta\theta\right)dx = 0\end{aligned} \tag{5}$$

where $(EA)_{com}$ denotes axial rigidity. $(EI_y)_{com}$ and $(EI_z)_{com}$ represent the flexural rigidities of the y- and z-axes, respectively. $(EI_\phi)_{com}$ and $(GJ)_{com}$ are, respectively, warping and torsional rigidities of composite thin-walled beams with doubly symmetric I-section, defined by [1]:

$$\begin{aligned}
(EA)_{com} &= 2A_{11}^f b + A_{11}^w d \\
(EI_z)_{com} &= 2A_{11}^f y_1^2 b + A_{11}^w y_3^2 d + A_{11}^f\frac{b^3}{6} + D_{11}^w d \\
(EI_y)_{com} &= 2A_{11}^f\left(\frac{d}{2}\right)^2 b + 2D_{11}^f b + A_{11}^w\frac{d^2}{12} \\
(EI_\phi)_{com} &= 2A_{11}^f\left(\frac{d}{2}\right)^2 b\left(y_1 - y_c - b\right)^2 \\
&+ A_{11}^f\left(\frac{d}{2}\right)^2 b\frac{b^2}{6} + 2D_{11}^f b\left(\left(y_1 - y_c\right)^2 + \frac{b^2}{12}\right) \\
&+ \left(A_{11}^w\left(y_3 - y_c\right)^2 + D_{11}^w\right)\frac{d^3}{12} \\
(GJ)_{com} &= 2\left(2D_{11}^f b + D_{11}^w d\right)
\end{aligned} \tag{6}$$

As mentioned earlier, in this study, the tolerable buckling loads are attained using the Ritz' method. To this aim, the shape functions including the torsion angle θ, the lateral deflection v, and the vertical deformation w for two different types of beams are chosen in the following forms:

Cantilevers with completely restrained warping at the fixed end [2]:

$$\{w(x) \quad v(x) \quad \theta(x)\} = \sum_{j=1}^{n}\{a_j \quad b_j \quad c_j\}\left(1 - cos\left(\frac{(2j-1)\pi x}{2L}\right)\right) \tag{7}$$

Simply supported with free bending and warping at both ends [2]:

$$\{w(x) \quad v(x) \quad \theta(x)\} = \sum_{j=1}^{n}\{a_j \quad b_j \quad c_j\}\left(sin\left(\frac{(2j-1)\pi x}{L}\right)\right) \tag{8}$$

Here, the terms a_j, b_j, c_j represent the undetermined Ritz coefficients.

3- Results and Discussion

To assess the effects of axial preloading on the buckling moment capacity of laminated composite double-tapered C-shaped beam-column element exposed to pure bending, a web and flanges tapered member with a span of L=2.4 m is considered. At the left end, the web of the selected beam is supposed to be 110 mm deep and both flanges are 70 mm wide, respectively. In the presence of a double-tapered element, the tapering parameters for the web and flanges are also considered to be $\beta = 0.4$ and $\alpha = 0.4$, respectively. Additionally, it is assumed that the web and each flange respectively consist of n_w=36 and n_f=24 fiber-reinforced composite layers, and the thickness of each ply is considered to be 0.25 mm. Based on this assumption, the whole thicknesses of the web section and each flange are respectively t_w=*6 mm* and t_f=*9 mm*. The material features for the fiber-reinforced

Table 1. The sequences of lamination for the web and both flanges of channel-section beam

No.	Top and bottom flanges	Web
1	$[0/90]_{18}$	$[0/90]_{12}$
2	$[45/0_4/45]_{3s}$	$[\pm 45]_{12}$

composite layers (glass-epoxy) are as follows, E_x = 75 GPa, E_y = 5.5 GPa, Gxy = 2.3 GPa, and υxy = 0.34.

Considering a prespecified stacking sequence (Table 1), the variation of the sustainable buckling moment of simply-supported as well as fixed-free C-shaped beam-column with respect to the compressive axial preloading (P^0) is depicted in Fig. 2.

4- Conclusions

Graphical results reveal that the fluctuation of the lateral stability strength with axial preloading is nonlinear for both simply-supported and cantilever channel-section beam-column elements. The extracted diagrams show that including the compressive axial force diminishes the buckling moment capacity for different axial load positions. The total deflection of the C-section member grows dramatically as the axial compression force approaches the critical load, resulting in a considerable drop in the buckling moment resistance of the selected member. Furthermore, all of the situations studied show that compressive axial force acting along the centroid provides the greatest resistance to lateral instability.

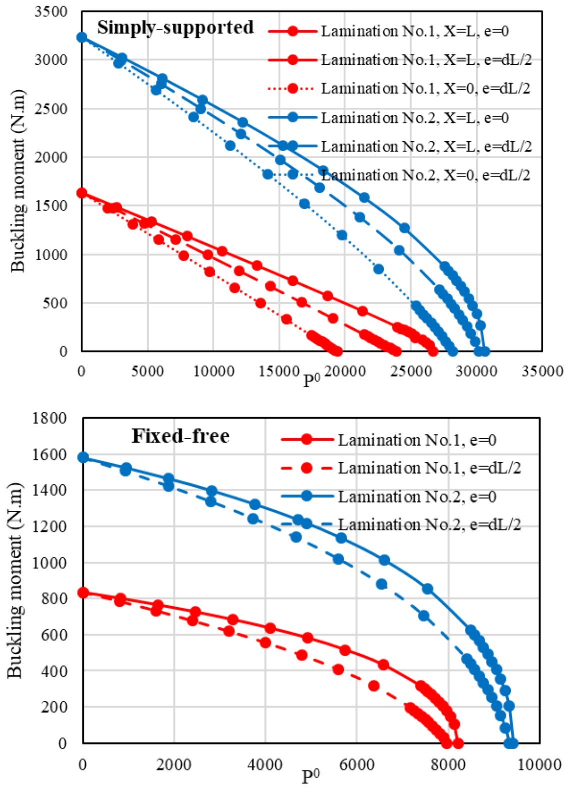

Fig. 2. Variations of buckling moment for laminated composite tapered C-shaped beam-column ( ) subjected to pure bending and compressive axial preloading for two different laminations (e: axial load eccentricity, d: the web height)

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