



## Analysis of the semi-elliptical crack in the FGM spherical pressure vessels

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**ABSTRACT:** One of the most important analyses in the design of various structures, especially pressure vessels and their safety is the fracture mechanics. One of the important parameters in fracture mechanics is the study of the stress intensity factor of cracks in the tank wall. In the present study, the behavior of a semi-elliptical crack in a spherical pressure vessel made of functionally graded materials has been studied using Abaqus finite element software. The effects of parameters such as crack geometry, simultaneous internal and external cracks, pressure distribution, thermal load distribution, changes in the properties of the functionally graded material, and support conditions on the value of the stress intensity factor have been investigated. To model and analyze the stress intensity factor in this type of tank, various power, exponential, and linear functions have been used in the form of MATLAB code as well as a subroutine code. Crack geometry is also an important factor that has a significant effect on the stress intensity factor. So with an increase in the  $a/c$  value, the stress intensity factor also increases. Also, the examination of the support conditions shows that with the increase in the number of foundations, the stress intensity factor also increases.

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**1- Introduction**

Pressure vessels are among the important structures that have special use in industries such as power plants, petrochemicals, oil and gas, and transportation, so the category of their design and construction is important. Spherical tanks have good strength and their pressure tolerance is much higher than cylindrical tanks. The first article published in the field of crack analysis in functionally graded materials was in 1992 by Noda and Jin [1] who showed that the correct selection of material constants reduces stress intensity coefficients. Eskandari [2] has studied the coefficient of stress intensity in a spherical tank under internal pressure and heat, with a functionally graded cover, using the exponential function, with three-dimensional finite element analysis. In 2020, Habibi and Bahrampour [3] estimated the stress intensity factor in a functional graduated spherical tank under pressure.

In this research, a spherical tank made of functionally graded metal-ceramic material (aluminum-zirconia alloy) with a semi-elliptical crack is modeled in Abaqus software. The mutual effects of two simultaneous internal and external cracks and support conditions (the effect of tank foundations) on the stress intensity factor have been investigated, which are not observed in previous research.

**2- Modeling and validation process**

The geometry considered in this research for both homogeneous and functionally graded materials is a spherical tank with an inner radius of 200 mm, an outer radius of 215 mm, a thickness of 15 mm, a crack tip radius of 4 mm, and an internal pressure of 20 MPa.

Equation (1) is used to calculate the stress in the thin-walled spherical tank. Equations (2 to 4) are also used to determine the stress intensity factor in semi-elliptical cracks under mode one loading in spherical pressure vessels.

$$\sigma = \frac{PR_m}{2t} \quad (1)$$

$$K_I = \frac{\sigma\sqrt{\pi a}}{\Phi} \left[ \sin^2 \theta + \left(\frac{a}{c}\right)^2 \cos^2 \theta \right]^{\frac{1}{4}} \quad (2)$$

$$\Phi = \int_0^{\frac{\pi}{2}} \left[ 1 - \left(\frac{c^2 - a^2}{c^2}\right) \sin^2 \theta \right]^{\frac{1}{4}} d\theta = \frac{\pi}{2} \left[ \frac{3}{4} + \left(\frac{a}{2c}\right)^2 \right] \quad (3)$$

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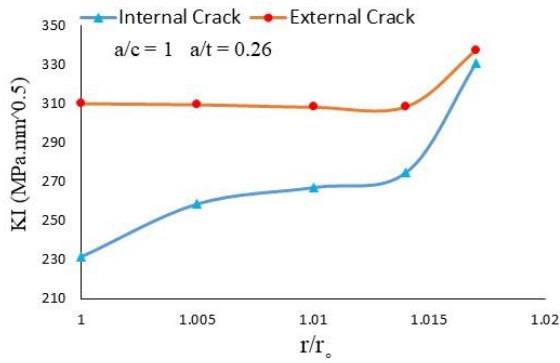


Fig. 1. Comparing the stress intensity factor in two internal and external cracks simultaneously in a tank

$$K_I = \frac{2\sigma\sqrt{\pi a}}{\pi \left[ \frac{3}{4} + \left( \frac{a}{2c} \right)^2 \right]} \left[ \sin^2 \theta + \left( \frac{a}{c} \right)^2 \cos^2 \theta \right]^{\frac{1}{4}} \quad (4)$$

Equation (5) is also used to calculate the stress intensity factor in the semi-circular crack.

$$K_I = \frac{2}{\pi} \sigma \sqrt{\pi a} \rightarrow (a = c) \quad (5)$$

In the above relationships,  $\sigma$  is the stress on the crack faces,  $P$  is the internal pressure,  $t$  is the thickness and  $R_m$  is the internal radius of the tank,  $a$  is the small radius and  $c$  is the large radius of the semi-elliptical crack and  $K_I$  is the stress intensity coefficient in mode one. [3]

For the accuracy of the modeling and analysis process, a homogeneous steel tank containing a semi-circular crack with a similar geometry was analyzed and the results were compared and validated with the analytical relations of fracture mechanics (Eq. 5). Then, the spherical tank containing three-dimensional semi-circular and semi-elliptical cracks is modeled with the same method and made of metal-ceramic functionally graded materials, the inner surface of aluminum metal, and the outer surface of zirconia ceramic. The stress intensity coefficient was extracted using the integral contour method throughout the crack front. The mathematical models of functionally graded materials used in this research to determine properties are power, exponential, and linear functions, which are defined according to equations (6 to 8), respectively.

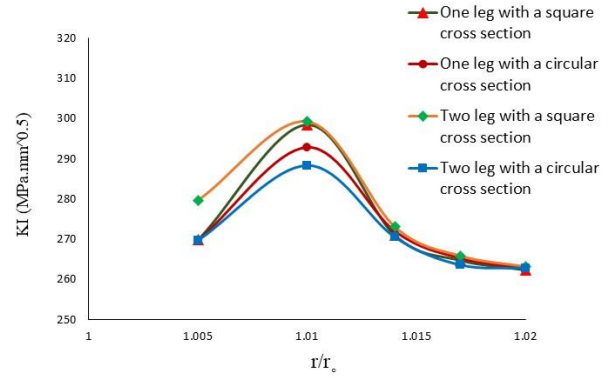


Fig. 2. Comparison of the stress intensity coefficient in the square and circular cross-section of one-leg and two-leg models in the quadrant of the tank containing transverse cracks

$$P = (P_c - P_m) \left( \frac{r - r_m}{r_c - r_m} \right)^n + P_m \quad (6)$$

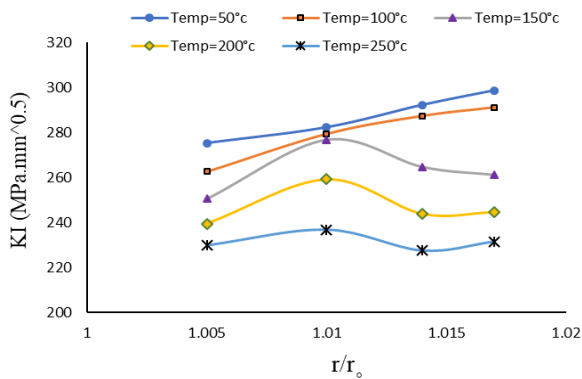
$$P = P_m \cdot e^{\gamma r}, \quad \gamma = \frac{\ln \frac{P_c}{P_m}}{(r_c - r_m)} \quad (7)$$

$$P = P_m (1 + (\gamma \cdot r)), \quad \gamma = \frac{\left( \left( \frac{P_c}{P_m} \right) - 1 \right)}{(r_c - r_m)} \quad (8)$$

The parameter  $P$  in the above equations (6 to 8) is the same as the properties of the functionally graded material, where  $P_m$  is the properties of the metal phase and  $P_c$  is the properties of the ceramic phase.  $r$  is the radius of the desired point,  $r_m$  is the radius of the metal phase (inner radius of the tank) and  $r_c$  is the radius of the ceramic phase (outer radius of the tank). In equations (7 and 8), the symbol  $\gamma$  is the constant of the formula, which depends on the properties and thickness of the tank. [4]

### 3- Results and discussion

In this section, to determine the points on the crack front, the ratio of the radius of each node of the crack front to the inner radius ( $r/r_0$ ) is considered. Figures 1 and 2 show the results of the stress intensity factor based on the effect of two simultaneous cracks and support conditions. The results revealed that in two simultaneous cracks with the same geometry and loading, the stress intensity factor in the internal crack is lower than in the external crack. Investigating the impact of the support conditions of the tank on the stress intensity factor also shows that increasing the



**Fig. 3. Comparison of thermal analysis results for different values of the internal surface temperature of the tank (°C) and constant pressure (P =20 MPa)**

number of foundations with each cross-sectional area also increases the stress intensity factor. Also, the comparison of the obtained results indicates that the base tank with a circular cross-section is more optimal. Figure 3 also shows the thermal analysis of the tank, taking into account the boundary conditions, for different temperatures on the internal surface and 25 °C on the external surface. From the results, it can be seen that as the surface temperature inside the tank increases, the stress intensity coefficient decreases.

#### 4- Conclusions

In the current research, by finite element analysis of the functionally graded spherical tank containing semi-elliptical cracks in three dimensions by Abaqus software, the stress intensity coefficient was analyzed and investigated. The results indicate that the external crack on the surface of the tank is more critical than the internal crack under similar loading conditions and at the same time. The results of thermal analysis also show the good thermal resistance of functionally graded materials.

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