

# Numerical analysis of moist-air flow in converging-diverging nozzle with equilibrium and non-equilibrium thermodynamic models

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## ABSTRACT

In this paper, the numerical solution of compressible, transonic, unsteady, inviscid and two-phase of moist-air flow in converging-diverging nozzles is studied. To do so, both equilibrium and non-equilibrium thermodynamic models with Roe's scheme are considered and the results are compared. In equilibrium thermodynamic model, the solver is spatially third order and temporally second order accurate, but in non-equilibrium thermodynamic model, the solver is spatially first order and temporally second order accurate. For the moist air in dry region the pressure, temperature, and velocity are extrapolated while in wet regions the steam quality has been used instead of pressure. In this study, the influence of the geometry expansion rate and inlet total temperature and pressure on nucleation rate and the wetness fraction at the nozzle exit are investigated. The results show that by increasing the expansion rate of nozzle the condensation onset occurs earlier; also, the nucleation rate and wetness fraction at the nozzle exit is increased. Comparing the results of equilibrium and non-equilibrium thermodynamic models show that the non-equilibrium thermodynamic model has better agreement with the experimental data.

## KEYWORDS

Roe's scheme, moist air, wetness fraction, equilibrium thermodynamic, non-equilibrium thermodynamic.

## Introduction

Condensation phenomena in pure steam or moist-air flows is one of the most important factors that can produce excessive losses and cause mechanical erosions for example in turbines or aircrafts. Therefore, the numerical scheme for prediction of moisture in condensing flows is essential. Various numerical studies have been done on the modeling of condensable pure steam and moist-air flows [1-3].

In this study, the numerical solution of condensing transonic moist-air flow through converging-diverging nozzle using equilibrium and non-equilibrium thermodynamic models is investigated and the results are compared. In non-equilibrium thermodynamic model which happens in reality (for transonic flow), when the super saturation ratio or supercooling level reaches a critical value, condensation occurs by spontaneous nucleation of tiny liquid droplets.

## Governing equations

The governing equations in equilibrium thermodynamic model for quasi one-dimensional, unsteady, inviscid and compressible flows are composed of the conservation laws of continuity, momentum and energy, and are shown in full conservative form. In the absence of body forces one can write [4]:

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} = H \quad (1)$$

$$Q = \begin{bmatrix} \rho A \\ \rho u A \\ \rho e_p A \end{bmatrix}, F = \begin{bmatrix} \rho u A \\ (P + \rho u^2) A \\ \rho u h_p A \end{bmatrix}, \quad (2)$$

$$H = \begin{bmatrix} 0 \\ P \frac{dA}{dx} \\ 0 \end{bmatrix}$$

$$\rho = \rho_{mix} = \rho_s + \rho_a, P = P_{mix} = P_v + P_a \quad (3)$$

Here  $Q$ ,  $F$  and  $H$  are respectively, the conservative vector, the flux vector and the source term.  $A$  is the cross-sectional area of the nozzle,  $\rho$  is the mixture density,  $u$  is the velocity,  $P$  is the mixture pressure,  $e_t$  and  $h_t$  are respectively the total internal energy and total enthalpy of the mixture.

The governing equations in non-equilibrium thermodynamic model for quasi one-dimensional flow in full conservative form consist of Euler equations for the mixture and four additional partial differential equations describing the condensate formation [5]:

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} = H \quad (4)$$

$$Q = \begin{bmatrix} \rho A \\ \rho u A \\ \rho e_t A \\ \rho g \\ \rho Q_2 \\ \rho Q_1 \\ \rho Q_0 \end{bmatrix}, F = \begin{bmatrix} \rho u A \\ (P + \rho u^2) A \\ \rho u h_t A \\ \rho g u \\ \rho Q_2 u \\ \rho Q_1 u \\ \rho Q_0 u \end{bmatrix}, \quad (5)$$

$$H = \begin{bmatrix} 0 \\ P \frac{dA}{dx} \\ 0 \\ 4\pi r_c^3 \rho_l J / 3 + 4\pi \rho \rho_l Q_2 \dot{r} \\ Jr_c^2 + 2\rho Q_1 \dot{r} \\ r_c J + \rho Q_0 \dot{r} \\ J \end{bmatrix}$$

Here  $Q_0$ ,  $Q_1$  and  $Q_2$  are Hill's moments,  $J$  is the homogeneous nucleation rate,  $g$  is the liquid mass fraction and  $\dot{r}$  is radius growth rate.

## Results and Discussion

To validate numerical study, the pressure distribution along nozzle is compared with the experimental data of Delale et al [6]. These comparisons indicate relatively good agreements. The maximum error in equilibrium thermodynamic model is obtained as 10%, while in the case of non-equilibrium thermodynamic model the maximum error is about 7%. Therefore, the results show that the non-equilibrium thermodynamic model has better agreement with the experimental data.

In this study, three nozzle geometries (A, B and C) taken from the Moore et al. [7] is considered. Nozzle A

of these series has the highest expansion rate while nozzle C has the lowest. Figure 2 shows the numerical results of wetness fraction along nozzles A, B and C using non-equilibrium thermodynamic model. The results show that, the condensation onset in nozzle A starts sooner than other nozzles, also, the wetness content at the exit of nozzle A is higher than that of nozzles B and C.

In Fig. 3 the profiles of droplet nucleation rate along the nozzles A, B and C is shown. As shown in this figure, due to the high expansion rate of nozzle A, in addition to wetness fraction (which is shown in Fig. 2), the droplet nucleation rate in nozzle A is more than that of nozzle B, similarly the nucleation rate in nozzle B is more than nozzle C.

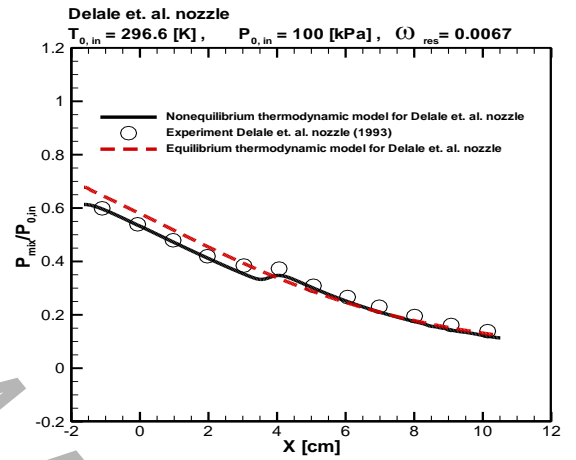


Fig. 1. Comparisons the numerical results of pressure distribution using equilibrium and non-equilibrium thermodynamic models with the experimental data of Delale et al. [6]

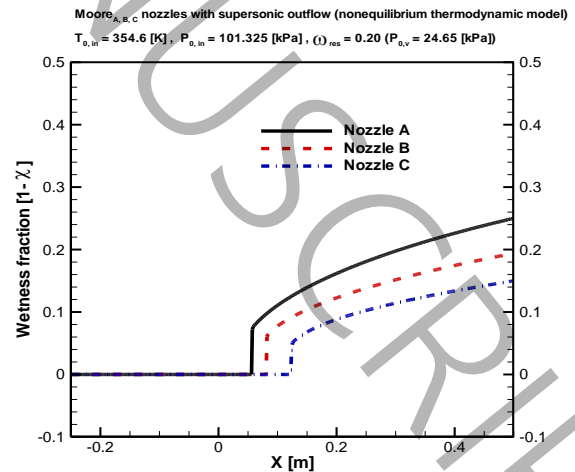
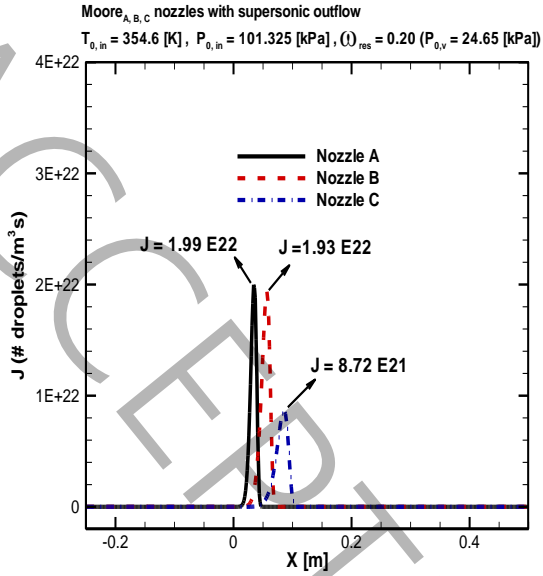


Fig. 2. Parametric study to illustrate the influence of nozzle geometry on condensation onset and outlet wetness fraction using non-equilibrium thermodynamic model



**Fig. 3. Droplet nucleation rate distribution along nozzles A, B and C using non-equilibrium thermodynamic model**

Table 1 shows the mass flow rate of produced water at the exit of nozzle A in equilibrium and non-equilibrium thermodynamic models for the condensing moist air flow. Therefore, this research can be considered for liquid water production from moist air flow in highly humid regions.

**Table 1. The mass flow rate of water at the exit of nozzle A in equilibrium and non-equilibrium thermodynamic model at the following conditions:  $P_{0,in}=101.325$  kPa,  $T_{0,in}=354.6$  K,  $\omega_{res}=0.2$**

model	$\dot{m}_l$ (kg/s)	$\dot{m}_l$ (kg/hr)
Equilibrium thermodynamic	0.291	1047.6
Non-equilibrium thermodynamic model	0.28	1008

## Conclusions

Condensation phenomena for transonic moist-air flow through converging-diverging nozzles using equilibrium and non-equilibrium thermodynamic model is studied numerically. The task is performed using a flux

difference splitting scheme of Roe. In equilibrium thermodynamic model, the solver is spatially third order and temporally second order accurate, but in non-equilibrium thermodynamic model, the solver is spatially first order and temporally second order accurate. The results of non-equilibrium thermodynamic model show better agreement with experimental data. In this study, the influence of geometry expansion rate and stagnation conditions on the flow field and wetness fraction are investigated. The results show that by increasing the expansion rate of nozzles, the condensation occurs sooner and the content of liquid water at nozzle exit increases.

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