



## Singularity, dynamics, and kinetics analysis of a 5 degrees of freedom parallel robot using screw theory

S. Khosravi<sup>1,2</sup>, M. Ghassabzadeh Saryazdi<sup>1\*</sup>

<sup>1</sup> Technology Institute of Mechanical Engineering, Amirkabir University of Technology, Tehran, Iran

<sup>2</sup> Mechanical Engineering Department, Amirkabir University of Technology, Tehran, Iran

**ABSTRACT:** This paper examines a parallel robot with 5 degrees of freedom with a linear platform. Parallel robots have a restricted workspace, and singularities make the workspace even more confined. So the behavior of the robot in the workplace is examined by focusing on kinematics and dynamics. To do kinematic analysis, the constraint equations are developed using the geometric relations, and the speed and acceleration equations of the robot are derived. The Jacobian matrix is then calculated using the screw theory, and the state of the singularities in the workspace is determined based on the Jacobian matrix. Considering the singularity and physical and geometric limitations, an algorithm for calculating the workspace is presented. In addition, the kinematic index of dexterity is investigated using the Jacobian matrix as a measure of the robot's closeness to the singular configurations. The results of solving kinematic and dynamic problems are validated with the output of the simulation in MATLAB software.

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## 1- Introduction

During recent decades, the machine tool industry has discovered the potential benefits of parallel mechanisms and many parallel mechanisms with 5 and 6 degrees of freedom structures have been developed [1]. Parallel robots have significant advantages in machining applications, especially the machining of complex surfaces, due to the ratio of their weight to the load capacity they bear, as well as their suitable orientations in space.

The current research robot is a 5-degrees-of-freedom parallel robot with 4SPRR-SPR configuration. In [2] Garrett et al analyzed the inverse kinematics of this structure. In [3], considering a robot with this structure, Boras et al. investigated the changes in the connection point of the robot's legs to the ground from the point of view of singularities. In [4], in addition to kinematic analysis, Guo et al analyzed the stiffness of this structure by applying the screw theory. Reference [5] is in the limited category of research that has dealt with the dynamic analysis of this structure, in which the dynamics of the robot has been extracted by the method of Kane's equations.

Screw theory, inspired by Newton-Euler relations, analyzes the dynamics of multibody systems in a matrix form, in addition to having the advantages of Lagrange equation and Newton-Euler method and high calculation speed, it

is efficient for multibody systems with a high number of members.

In this article, the inverse kinematics calculations of the robot are first performed, then the Jacobian matrix is extracted using the screw theory. All singular orientations of the robot are obtained at every point of the working space. The kinematic index of dexterity is presented as a measure of the proximity of the robot to its singular points in the workspace. The dynamics of the robot are analyzed using the screw theory. Finally, all the obtained results are validated by simulating in MATLAB's sim-mechanics environment.

## 2- Modeling

The five-degree-of-freedom robot of the current research has a 4SPRR-SPR structure, Figure 1 shows how to connect the legs of the robot to the end effector and the fixed base.

The screw axes of the robot joints are shown in Figure 3. In equation (1), the Jacobian matrix of the robot is written, which is obtained based on the screw theory

$$J = \begin{bmatrix} ON_1 \times S_{1,4} & S_{1,4} \\ ON_2 \times S_{2,4} & S_{2,4} \\ ON_3 \times S_{3,4} & S_{3,4} \\ ON_4 \times S_{4,4} & S_{4,4} \\ ON_5 \times S_{5,4} & S_{5,4} \\ OM_1 \times S_{1,1} & S_{1,1} \end{bmatrix} \quad (1)$$

\*Corresponding author's email: mghsaryazdi@aut.ac.ir



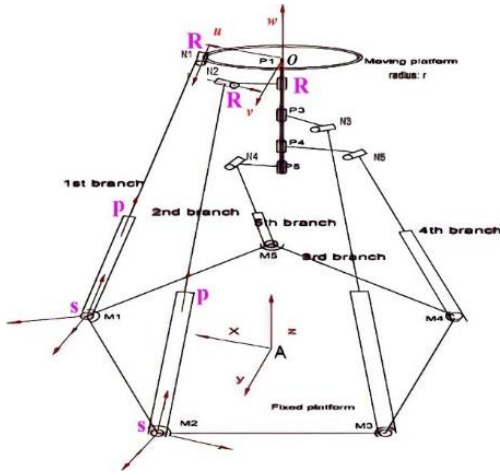


Fig. 1. 5 degrees of freedom parallel robot configuration

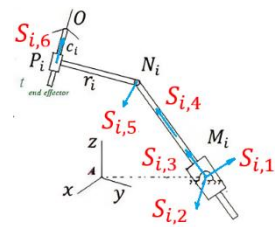


Fig. 3. Joint screw in the robot

In screw theory, the dynamic relationships of each member are written separately, and by assembling all the equations, matrix relationships are obtained to extract the forces of operators and reaction forces.

$$t_i = [\omega_i^T \ V_i^T]^T, \ t = [t_1^T \ t_2^T \ \dots \ t_{n-1}^T]^T \quad (2)$$

$$\begin{bmatrix} K^T & T^{Ac} \\ S^T & o \end{bmatrix} \begin{bmatrix} \lambda \\ \tau \end{bmatrix} = \begin{bmatrix} \eta \\ 0 \end{bmatrix} \quad (3)$$

$$\eta = Mi + WMt - w^s \quad (4)$$

where the matrix  $K$  with dimensions  $6n \times 6(n - 1)$  contains the constraint coefficients of all kinematic pairs, which is called the robot constraint matrix.  $t$  is a  $6(n - 1)$  vector that includes the twist of all links of the robot.  $w$  is the wrench vector of constraint torques and forces, also  $T^{Ac}$  called the actuator wrench shaping matrix.  $M$  is the mass matrix of the robot and  $W$  is the angular velocity matrix of the robot,  $w^s$  is the vector of external forces acting on the robot.

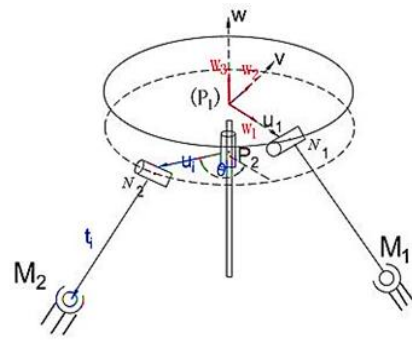


Fig. 2. Schematic of the end effector

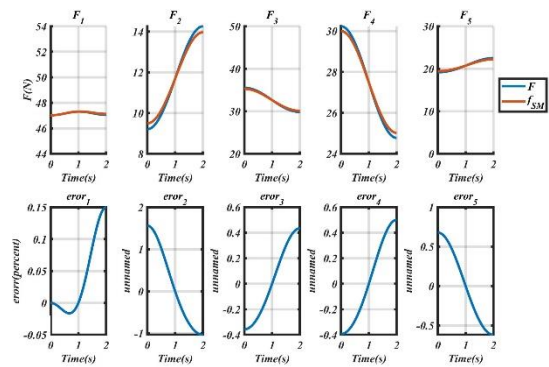


Fig. 4. The driving force the actuators and the error percentage with the simulation results

### 3- Results and discussion

The results of the analytical model based on the screw theory and sim-mechanics simulation are presented in Figure 4.

The physical interpretation of orientations in which the determinant of the Jacobian matrix becomes zero and the robot is placed in a singular state is shown in Figure 5.

The working space of the robot is obtained for any orientation, taking into account the constraints of equation (10). the workspace for a specific orientation is illustrated in Figure 6.

$$|J| \neq 0, \ L_{i_{min}} \leq L_i \leq L_{i_{max}}, \ 5^\circ \leq \pi - \beta_i \leq 85^\circ \quad (5)$$

The kinematic dexterity index is defined as a measure of how close the robot is to its singular configurations.

$$DI = \frac{\sigma_{min}}{\sigma_{max}} \quad (6)$$

where  $\sigma_{min}$  is the smallest singular value of the Jacobian matrix and  $\sigma_{max}$  is its largest singular value. Figure 7 shows

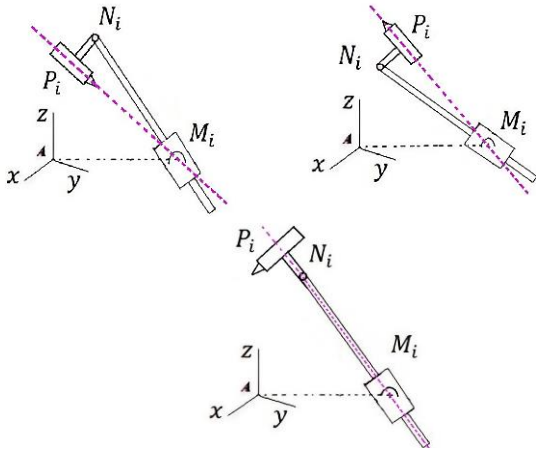


Fig. 5. Robot orientation in a singular configuration

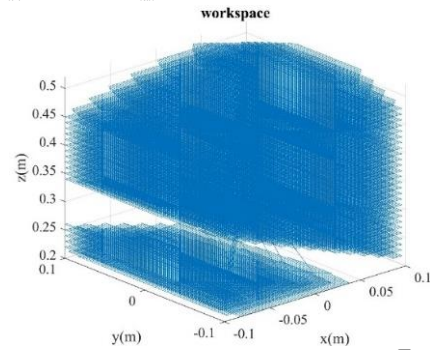


Fig. 6. Workspace for  $\alpha = 0, \gamma = \pi/6$

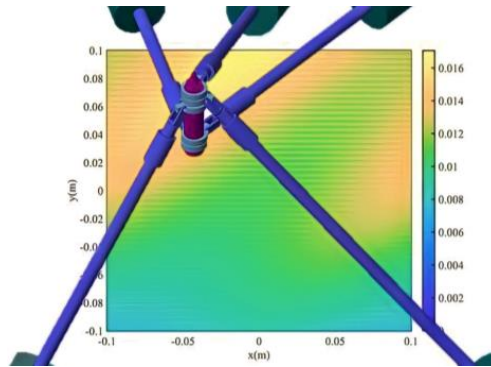


Fig. 7. Dexterity index distribution in a square space at a height of  $z=0.25m$ ,  $\alpha = 51^\circ$

the contour of the robot's dexterity for an optimal position and orientation.

#### 4- conclusion

The aim of this research is the kinematic and dynamic analysis of a parallel robot. The Jacobian matrix and the dynamic equations of the robot were derived using the screw theory. By calculating the Jacobian matrix, singular points of the robot were extracted and by applying physical constraints, an algorithm was presented to calculate the working space of the robot. Then, the dexterity index was presented as a measure of the distance of the robot from singular points. This index was checked in the working space of the robot and it was determined that the robot has the highest value of dexterity index at the height of  $z=0.25m$  and  $\alpha = 51^\circ$ .

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