

# A novel objective function for path generation synthesis of the four-bar mechanism with prescribed timing

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## ABSTRACT

Dimensional synthesis of the four-bar mechanisms, as the simplest and most widely used linkage, is always one of the main issues raised in the field of mechanism design. In the past, geometric and graphical analysis methods were used to design these mechanisms, but with the advancement of technology in recent decades, numerical optimization methods and meta-heuristic algorithms have been utilized more. Moreover, it is possible to solve problems more precisely and consider a higher number of precision points. The path generation synthesis problem of this mechanism has been solved using different algorithms and in different design modes, including with and without prescribed timing. The synthesis problem with prescribed timing is more difficult and has a higher path generation error. In this research, to solve the path generation synthesis problem of the four-bar mechanism with prescribed timing, a novel objective function is utilized. The proposed function includes two terms, path error and angle error. This new objective function leads to a lower error for path generation with prescribed timing. Four different designs are considered and their results, where extracted by the PSO algorithm, are compared. The results of solving this problem for three numerical examples show that the design in the proposed way, has fewer path generation errors.

## KEYWORDS

Four-bar mechanism, Path generation synthesis, Prescribed timing, Particle swarm optimization algorithm.

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## Introduction

Mechanisms, as basic industrial elements with the ability to accurately control movements and transfer forces, help to improve quality, and efficiency, reduce costs, and increase the safety of industrial production. In this regard, the accurate and optimal design of mechanisms is considered essential in modern industry. Designing mechanisms for different applications and producing different movements are always difficult but important issues. In the design of mechanisms, only for a certain and limited number of accuracy points, the mechanism can be designed in such a way that it passes through all those accuracy positions accurately (without error) [1, 2]. For problems with a higher number of accuracy points, methods such as the least squares method or optimization methods should be used [3, 4]. Four-bar mechanisms are a group of simple mechanisms and in the division of mechanisms based on the working space, these mechanisms are considered part of the plane mechanisms. The main goal of this paper is the optimal design of a four-bar mechanism based on the route generation problem and by providing a new objective function. To reach the desired objective function, four different design modes are considered and their results, which were extracted using the particle swarm optimization algorithm, are compared.

## Methodology

Fig. 1 shows the general and detailed schematic of the angles and links of the four-bar mechanism, where the ground member is link number 1, the intermediate member is link number 3, and the input and output members are links number 2 and 4, respectively. According to the figure,  $r_1$  is the length of the fixed member (ground),  $r_2$  is the length of the input member,  $r_4$  is the length of the output member, and if the intermediate member is triangular,  $r_3$ ,  $r_5$ , and  $r_6$  are the lengths of the three sides of the intermediate member. Obviously, if the angular position of all members is determined, then the location of all points on the links, including the location of point P, can be obtained.

As depicted in Fig.1, the design variables for a prescribed timing synthesis problem of the four-bar mechanism are as follows:

$$X = [l_1, l_2, l_3, l_4, l_{5x}, l_{5y}, x_0, y_0, \theta_1] \quad (1)$$

Moreover, for the synthesis of the path generation of a four-bar mechanism in general mode, the objective function of the optimization problem, which is the error between the path generated by the mechanism and the desired (desired) path, is defined as follows:

$$F_1 = \frac{1}{n} \sqrt{\sum_{i=1}^n \left[ \left( C_{xd}^i - C_x^i(X) \right)^2 + \left( C_{yd}^i - C_y^i(X) \right)^2 \right]} \quad (2)$$

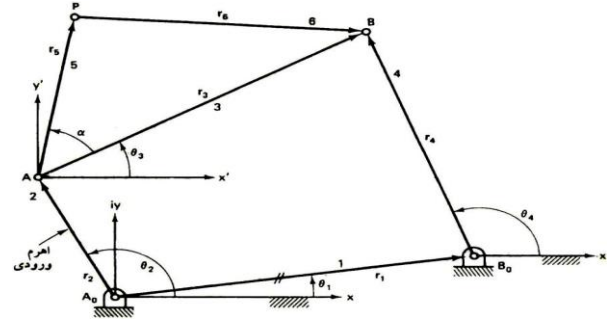


Figure 1. General schematic of the four-bar mechanism

Note that for a design problem without prescribed timing, the unknown values of the input link must be added to the design variables.

In the synthesis of the path production of a four-bar mechanism, different cases can be considered. In this article, the four design cases are defined as follows:

- Without prescribed timing:

In this case, no specific timing is determined for the mechanism to pass through the desired points, so the mechanism is free to pass over the desired points with at time interval.

- With prescribed timing:

In this case, although the main goal is still to design a four-bar mechanism based on the minimum path error, the timing of passing through the desired points is also important and its values are given by the problem.

- With prescribed timing ( $\theta_2^1$  is a design variable):

This mode is similar to the previous mode and the only difference is that  $\theta_2^1$  is part of the design variables in this mode. Here, the time difference in passing through different points is important.

- Design based on two objective functions:

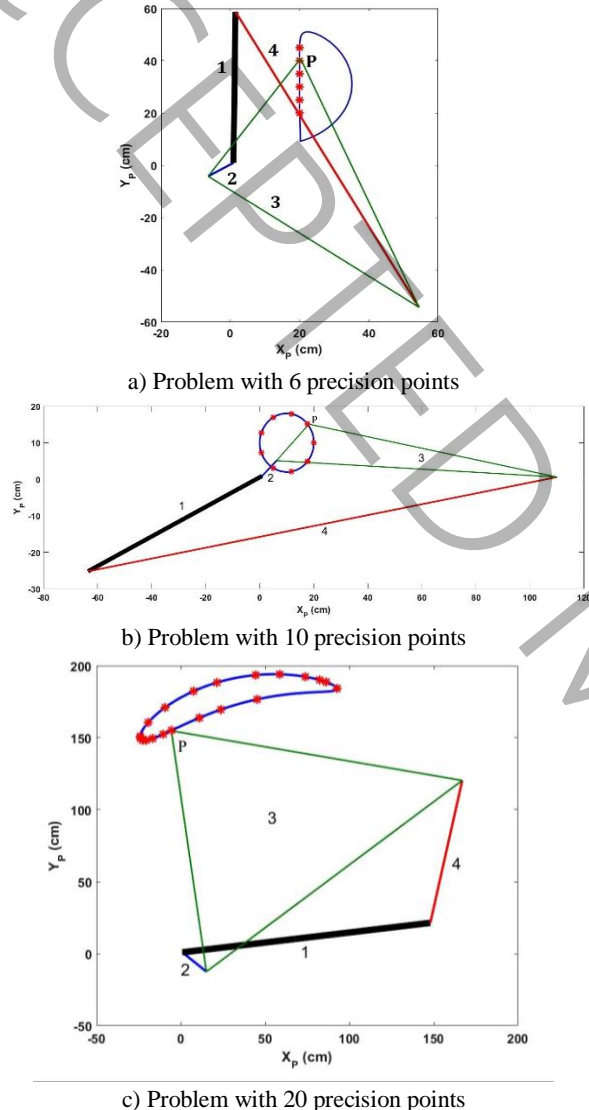
In this case, a prescribed timing design is considered by defining an acceptable range for the angles of the input link (corresponding to each accuracy point). Thus these angles can have changes in this range, and their values with respect to desired ones define the second error function. Therefore, in this case, both the path error and the angle error must be minimized. Finally, the two objective functions are added together and the overall objective function for this state will be as follows:

$$F_{Obj} = M_1 F_1 + M_2 F_2 \quad (3)$$

$$F_2 = \frac{1}{n} \sqrt{\sum_{i=1}^n (\theta_{2d}^i - \theta_2^i)^2} \quad (4)$$

## Results and Discussion

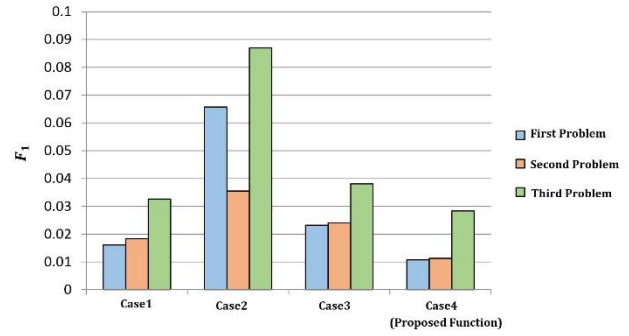
Fig. 2 shows the results based on the proposed objective function (Eq. 4) for different problems.



**Figure 2. Designed mechanisms based on the proposed objective function**

Moreover, Fig.3 shows a comparison between different design states in three numerical examples with 6, 10, and 20 precision points, respectively. The general results that can be extracted from designs in different situations and examples are:

- The path generation synthesis problem with a prescribed timing (case2) always has the highest error.
- Using the proposed function, the path error of the problem with a certain timing can be reduced and is the lowest among all cases.



**Figure 3. Comparison of the path error ( $F_1$ ) in different cases and examples**

## Conclusion

The optimal path generation synthesis of the four-bar mechanism for a prescribed timing was the main goal of this research. Synthesis problems with definite timing always have a higher path generation error than problems without definite timing. In this research, by presenting a new objective function, which includes both the path error and the orientation (angle) error, it has been tried to improve the design process. To prove the efficiency of this new function, three numerical examples have been used with 6, 10, and 20 precision points, respectively. PSO algorithm is also used for the optimization process. The design results in different situations show that the new objective function has been able to significantly reduce the path generation error for the problem with prescribed timing.

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