Formulation and Topology optimization of flexure joints with small deformations based on strain energy criteria

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ABSTRACT

Flexure joints are one of the most widely used and crucial elements in the design of precision mechanisms. Owing to their monolithic and elastic structure, these joints facilitate highly precise movements. In this study, we present a kinetoelastic model for designing various types of flexure joints with single and multiple degrees of freedom. To reduce computational costs, two beneficial approaches for defining the objective function and constraints are presented, based solely on the strain energy criterion and predetermined displacements. The resulting self-adjoint optimization problem exhibits computational efficiency and improved convergence. The topology optimization problem utilizes the Finite Element Method and the Solid Isotropic Material with Penalization model, employing the Method of Moving Asymptotes to solve and identify the optimal topology. A comprehensive mathematical framework, including the relevant two-dimensional boundary conditions and sensitivity analysis, is meticulously developed and extensively examined. For this purpose, MATLAB code is developed for designing two-dimensional flexure joints with single and multiple degrees of freedom. Finally, the results obtained from the comparison of two optimization approaches presented in this study are discussed. In these joints, the stiffness ratio of the structure has increased significantly, up to 208 times, indicating the practicality and effectiveness of this method in the topology optimization of flexure joints.

KEYWORDS

Flexure joints, topology optimization, strain energy, the Method of Moving Asymptotes, predetermined displacements

1. Introduction

Topology optimization is a branch of structural optimization that focuses on determining the optimal structure by adjusting the number, location, and shape of voids, and the way members of the structure interact. It's used in continuous structural problems [1] and designing flexible mechanisms like flexure joints, critical in engineering and robotics [2].

Flexure joints allow relative motion via elastic deformation, needing minimal maintenance due to their monolithic build and lack of internal friction [3]. The design of flexure joints must ensure that these mechanisms create specific relative motions between rigid links while maintaining desirable stiffness. Degrees of freedom enable desired movement, whereas constraints restrict it [4].

This research applies topology optimization to enhance flexure joint performance, emphasizing stiffness optimization, motion range, stress control, fatigue resistance, and manufacturing sensitivity. It focuses on joints with small displacements, using linear elasticity for structural relationships.

2. Methodology

A flexure joint within a design domain Ω made of linear elastic isotropic material is considered. The design domain is represented as in Figure 1 and includes a white optimization region and blue rigid links on the left and right. The contact areas between the flexure joint and the rigid links are called interfaces. Circles indicate common nodes where predetermined displacements apply various motion patterns to the joint.

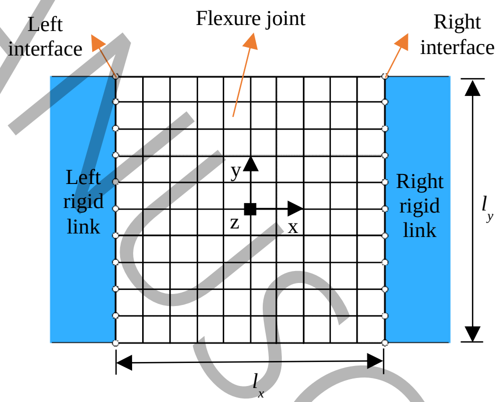

Figure 1. The design domain for topology optimization of flexure joints in a 2D space

This research uses predetermined displacements as boundary conditions, unlike previous studies that used fixed loads. In a 2D space, the motion patterns $\mathbb{M}=\{t_x,t_y,r_z\}$ include two translational (x and y axes) and one rotational (z-axis) movements, as shown in Figure 2. Subsets $\mathbb{C}\subset\mathbb{M}$ and $\mathbb{F}=\mathbb{M}\setminus\mathbb{C}$ represent constrained and free motion patterns, respectively. Table

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1 details the numbering and nodal displacement values for the three defined motion patterns.

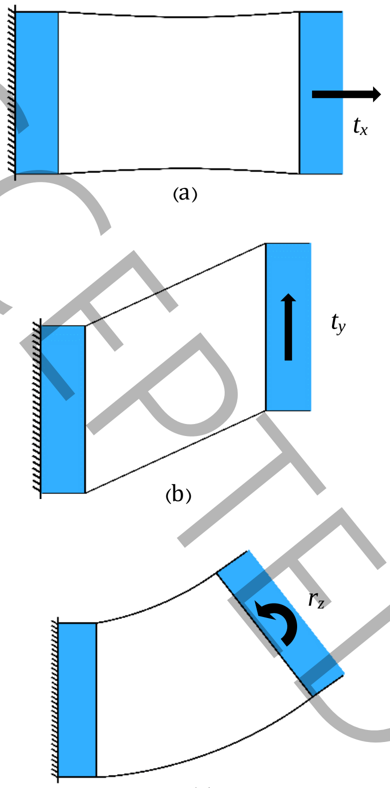

Figure 2. Three independent motion patterns used for the design of 2D flexure joints. (a) Relative translational motion along the x-axis (t_x), (b) Relative translational motion along the y-axis (t_y), and (c) Rotation about the z-axis (r_z).

Table 1. predetermined displacement values

Motion pattern	Number	u	v
t_x	1	1	0
t_y	2	0	1
r_z	3	1	$u=-\dfrac{2y}{l_y}$

Two methods for formulating the topology optimization problem are presented below. The first approach formulates the topology optimization problem by maximizing the strain energy function for constrained motion patterns while assigning specific values to the strain energies from free motion patterns. This function's negative is minimized for standard optimization.

In the second approach, a function is derived from strain energies due to applied motion patterns for both constrained and free degrees of freedom. The objective is to maximize constrained motion pattern energies and minimize free motion pattern energies simultaneously, while maintaining a specified minimum ratio between the energies of constrained and free motion patterns. Like the first approach, the optimization problem is formulated in a standard manner rather than maximizing constrained motion pattern energies directly. The formulation of the proposed problem is as follows:

$$\begin{aligned}
&\text{find } \boldsymbol{\rho} = \left[\rho_1 \quad \rho_2 \quad \ldots \quad \rho_n\right]^{\mathrm{T}} \\
&\min\ \mathrm{f}(\mathrm{U}(\boldsymbol{\rho})) = \sum_{i=1}^{n_c} -\mathrm{U}_i(\boldsymbol{\rho}) + \sum_{j=1}^{n_f} \mathrm{U}_j(\boldsymbol{\rho}) \\
&\text{s.t. } \frac{\mathrm{U}_i(\boldsymbol{\rho})}{\mathrm{U}_j(\boldsymbol{\rho})} \geq e_{\min}, \quad i \in \mathbb{F},\ j \in \mathbb{C} \\
&\qquad V(\boldsymbol{\rho}) = \sum_{k=1}^{n} \boldsymbol{\rho}_k^{\mathrm{T}} \mathbf{v}_k - \bar{V} \leq 0
\end{aligned} \tag{1}$$

The dimensionless objective function f depends on strain energies U_i and, U_j with e_{min} indicating the minimum relative stiffness ratio between constrained and free degrees of freedom's strain energies. V denotes the permissible material volume.

Topology optimization involves calculating successive structural responses (objective function and constraints) and analyzing their sensitivity to design variables. In this research, the optimization problem aims to minimize negative strain energy in constrained degrees of freedom. Consequently, the sensitivity analysis of the objective function in this optimization problem is expressed as follows:

$$\mathrm{F} = -\mathrm{U}_a \rightarrow -\frac{\partial \mathrm{U}_a}{\partial \rho_e} = -\frac{1}{2}\mathbf{u}^{\mathrm{T}}\frac{\partial \mathbf{K}}{\partial \rho_e}\mathbf{u} \tag{2}$$

3. Results and Discussion

In this section, an example of a rotational flexure joint along with its validation is presented. A rotational flexure joint can only rotate about the z-axis and has no translational motion along the x and y axes. Sets of free and constrained motion patterns, denoted as $\mathbb{F}=\{r_z\}$ and $\mathbb{C}=\{t_x,t_y\}$, respectively, are considered. The minimum relative stiffness of the constrained degrees of freedom relative to the free degree of freedom is considered to be 50 $(e_{\min}=50)$. The optimized topology depicted in Figure 3(a) represents the output of the optimization problem, and for a better comparison, the topology generated by Koppen [5] is presented in Figure 3(b).

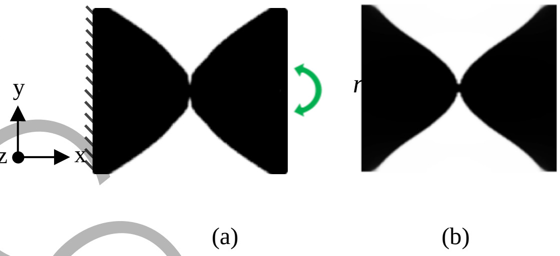

Figure 3. Optimized flexure joints capable of rotation about the z-axis resulting from (a) current research and (b) the design proposed by [6]

For the validation of the quantitative research conducted, the stored strain energy in two rotational hinge joints depicted in Figures 3(a) and 3(b) is calculated under the application of translational movement patterns t_x and t_y with identical inputs. Under these uniform conditions, the hinge joint derived from the approach presented in this study preserves 10.83% and 1.85% more strain energy for the constraint degrees t_y and t_x, respectively, compared to the hinge joint proposed by [6]. Convergence plot illustrating t_y comparison is shown in Figure 4. The output results of these two optimization problems are presented in Table 2.

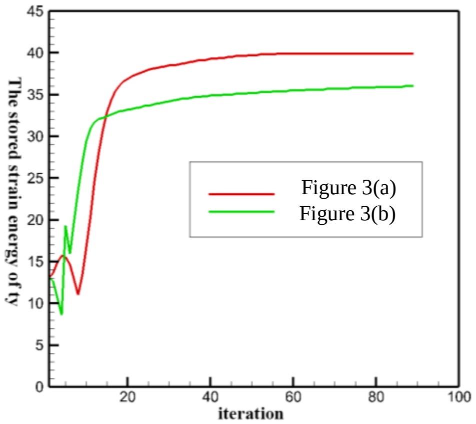

Figure 4. Comparison of convergence plot of strain energy t_y values for the rotational joints presented in Figures 3(a) and 3(b)

Table 2. Strain energy values stored in rotational joints of Figures 3(a) and 3(b)

Optimized flexure joint	(N.mm)		
	t_x	t_y	r_z
Figure 3(a)	110	39.9	0.851
Figure 3(b)	108	36	0.851

The validation confirmed the accuracy of finite element calculations and the optimization algorithm. A computer program was then developed to identify more complex rotational flexure joints. These optimized topologies reduce material at their centers, enhancing resistance to torsional loads and enabling rotational movement around their centers when fixed on one side and subjected to rotational force on the other.

The optimization method proposed in this study, based on strain energy criteria, and has significant advantages. The first advantage is its self-adaptive nature, which increases the solution speed and reduces computational volume to the extent that it can be executed on a home computer. The second advantage is its capability to generate numerous optimal topologies, whereas previous methods only produced a single optimal topology. The third advantage lies in the use of a gradient-based optimization approach, which facilitates rapid convergence of the objective function and constraints, thereby reducing the time to achieve optimal topologies.

4. Conclusions

This research establishes a comprehensive framework for the design of flexure joints, significantly simplifying the process by requiring minimal parameters and computational effort. Utilizing MATLAB and gradient-based methods, this framework effectively optimizes both single and multi-degree-of-freedom joints within the linear elastic range. Notably, some designs achieved a stiffness-to-weight ratio of up to 208 times. The framework also has potential applications in three-dimensional and nonlinear elastic problems, and can incorporate constraints such as stress and fatigue, making it highly practical and versatile for future advancements in the field.

5. References

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