

# Study on Angles-only Initial Orbit Determination methods and development of a method suitable for optical observation for LEO

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## ABSTRACT

The need to determine the orbit of satellites is inevitable, this orbit determination is done by signal, radar or optical methods, which according to the country's capabilities for this issue, we turned to the optical method, or in other words, Angles-only methods. In this article, we examined a variety of methods for Angles-only Initial Orbit Determination and finally found the Gaussian method suitable for our purpose. The Gaussian method has different approaches to the answer, and we developed an orbit determination with a new approach that aimed to eliminate the estimates, and finally presented an extended method without any estimates. To test this method, a two-body simulation was performed and its error-free results indicate the correct development and accurate implementation of the method. Also, for the case with complete Perturbation and noise more than 3arcsec, in comparison between this method and other classical methods and other articles, we find that the accuracy of this method for IOD with optical observation has better results or at the level of new IOD methods.

## KEYWORDS

Orbit determination, Initial orbit determination, Angles-only methods, Gauss method, Optical observation,

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## Introduction

To determine the initial orbit, there are different methods according to different inputs, and according to the available facilities in the country, in this article, we determine the orbit using only the angular method, in which three observation points are needed to determine the initial orbit; These three points are the beginning, middle and end points of the observation interval. The values of azimuth and elevation angles are obtained from optical imaging and calibrating the image with stars (like what happens in a star sensor and its elaboration is not included in this article). The output of this method is the speed and location vectors of the satellite at the midpoint of the observation, which can be used as the initial conditions for determining the exact orbit. The disadvantage of optical observation is the limited viewing angle in the sky, which makes it difficult to determine the general orbit (due to the limited visibility and short time to determine the orbit, as well as the need to know the initial location of the satellite to cross the sky). The advantage of the purely angular method of optical observation is the low cost of installation and use, in addition to its high accuracy compared to other methods of determining the orbit using a satellite pass.

In this orbit determination, the first task is to prepare observation and reference images using stars, the output of which is a table of inertial vector angles from the ground observer's location (J2000 topocentric). We did a conversion step from these angles to azimuth and elevation angles in the ecef. The next step is to generate the initial orbit - as the initial conditions for determining the exact orbit - from the beginning, end and middle points of observation. In this article, we examined this section, i.e., the types of primary orbit determination. In the next step, by changing this initial condition, we must produce a circuit that passes through all observation points. After this step, we will determine the exact orbit from all observations.

## Methodology

Optical observation to determine the orbit is limited to one pass due to the limitations of the necessary conditions in terms of exposure of the satellite to the ground station, and therefore the length of the pass does not exceed 10 minutes for low-altitude orbits; Due to the small angle between observation points and also the need to be resistant to noise greater than 5 arcsec for observation, we chose the Gaussian method. The Gauss method is divided into two parts, the first part of which is the conversion of three pointing vectors into location vectors and the second part is the conversion of location vectors into velocity.

The problem of two vectors of location and time of flight between them is usually known as Lambert's problem; Because he was the first person to form the solution [1]. Of course, Euler had solved this problem for parabolic orbits before him, and then Lambert generalized it for elliptic and hyperbolic orbits [2]. For this reason, in reference [2], the corresponding solution is presented under the name of Lambert-Euler. However, what we use is the Gaussian solution for this problem; Because after the comparison made in reference [1] between the different solutions proposed in it, the Gaussian method is optimal for location vector observations with an angle difference of less than 70 degrees.

The Gaussian method has different approaches based on the two-body problem, the approach that is mainly introduced in reference books is the use of Lagrange coefficients  $f$  and  $g$  [2], which in calculating the coefficients of the distance vector behaves with estimation and simplification; Therefore, the final solution of the Gauss method (what we will see in the comparison section of this article) is far from the original solution, which shows the simplification during the development of the method. In this section, we develop a method based on Gauss and Lambert's approach (transforming three pointing vectors into location vectors and using these location vectors to generate the velocity vector).

Considering that Lambert's problem does not have a direct analytical solution that accurately determines the orbit; The solutions provided for it are based on repeating a loop until the convergence of one variable is reached. With this explanation, the Gauss method is formed based on the ratio of the area of the orbit sector between the two location vectors ( $S$ ), to the area of the triangle formed by the two location vectors ( $\Delta$ ).

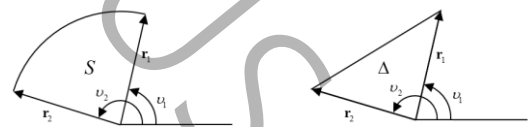


Figure 1. Definition of variables  $S$  and  $\Delta$

## Results and Discussion

To validate the process, we produced two problems, one without orbital perturbations (two-body problem) and the other with complete orbital perturbations in STK software, and access for 4 cities of Tabriz, Mashhad, Shahrood and Bandar Abbas with the requirement of a minimum elevation angle We also considered 15 degrees. We also added a measurement error value of

0.001 deg as noise to the data. The result of determining the orbit for each city (with the first, middle and end of the passage data) for both modes is as follows:

**Table 1. Comparison between the methods of initial orbit determination (two-body)**

Improved Gauss method (m)	double-r method (m)	Gauss method (km)	ground station
16	17	106	Mashhad
17	18	37	Tabriz
597	594	148	BandarAbas
20	21	55	Shahrood

**Table 2. Comparison between the methods of initial orbit determination (with perturbations)**

Improved Gauss method (m)	double-r method (m)	Gauss method (km)	ground station
970	970	107	Mashhad
980	980	38	Tabriz
1390	590	148	BandarAbas
680	680	56	Shahrood

We can see that the Gaussian method is much more accurate after modification. On the other hand, the accuracy of the Gauss and double-r method is the same, but in most cases it is more for the written code; But each method, according to what was mentioned in the previous sections, has its own advantage and is used in a specific type of orbit determination.

To compare with the articles, we need to solve their sample problems. In reference [3], the problem of determining the orbit is also solved from the two methods of Gooding and improved Gooding, which is the result of the reference article [3], and we calculated the classic orbit parameters with the values in the article and we have this results:

**Table 3. Comparison between keplerian elements**

	a (km)	e	i	$\omega$	$\Omega$	$\nu$
Gooding	9	1e-3	1e-5	0	0	0
Optimized Gooding	6	1e-4	1e-4	0	0	0

Improved Gauss method	3.7	1e-5	1e-3	0	0	0
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From Table 3, we can see that the accuracy of determining the orbit developed in this article is higher than both methods mentioned in reference [3]. Since the determination of the orbit is not only to obtain the location of the satellite and the speed is very important due to the need to predict the future position of the satellite, the influence of the main semi-diameter and eccentricity parameters is higher than the rest of the parameters, and with the more appropriate estimation of these two terms, the accuracy of determining the orbit for the application It is a better operation.

## Conclusions

We said that determining the initial orbit is only an initial value for exact solution with methods such as least squares or Kalman filter; The more accurate this initial orbit determination is, the exact orbit determination converges in fewer steps. If the initial orbit determination has a large error, it can cause the exact orbit determination to diverge. In a complete orbit determination system, which consists of two parts: initial orbit determination and exact orbit determination, the majority of the solution time is related to the exact orbit determination, which can help reduce the final solution time by focusing on the initial condition. On the other hand, the results of determining the primary orbit are based on the two-body problem; Therefore, in order to obtain a suitable orbit based on the Gaussian method, we also performed studies and solutions by eliminating simplifications and estimates, and finally we reached the accuracy of determining the orbit to a suitable level. We have greatly improved the accuracy of determining the orbit compared to the Gauss method, and it has reached the level of other modern methods, and we have obtained better results in many examples. The advantage of this solution method compared to other methods is the direction of the error vector, which for the developed method is only the direction of the distance vector.

## References

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