

# Parametric Study of Model-Based Dynamic Control Methods for Enhancing Locomotion in Underactuated Biped Robots, Case study: Hybrid Zero Dynamics and Proportional-Derivative Feedback

Roozbeh GhanadiAzar <sup>1</sup>, Mohammad Reza Haghjoo <sup>1\*</sup>, Mostafa TaghiZadeh <sup>1</sup>

<sup>1</sup> Faculty of Mechanical and Energy of Shahid Beheshti University (SBU), Tehran, Iran

## ABSTRACT

The parametric study of model-based dynamic control methods holds significant importance in biped robot motion control. This research delves into a detailed examination of the parameters of model-based dynamic control methods, specifically the Hybrid Zero Dynamics (HZD) and Proportional-Derivative (PD) feedback control methods, to enhance the locomotion of underactuated biped robots. A three-link underactuated biped robot without a knee joint with three degrees of freedom is used as a case study, and the dynamic equations for this model are extracted in continuous and impact phases. Robot simulations are executed in MATLAB software by comparing and analyzing the control parameters in the two mentioned methods, and the results are compared and discussed. Furthermore, the effect of variations in control parameters in the Proportional-Derivative feedback control method is evaluated and compared. The results indicate that the Hybrid Zero Dynamics method generates more symmetrical and uniformly paced movements than the Proportional-Derivative feedback control method, with lower control effort. Increasing the control parameters in the Proportional-derived feedback control method brings its results closer to those of the hybrid zero dynamics method, accompanied by a reduction in control effort. In addition to presenting results, this study meticulously examines and analyzes control parameters, which can enhance bipedal robot performance.

## KEYWORDS

Biped robot, walking dynamic stability, Based Dynamic Control, Proportional-Derivative feedback, Hybrid Zero Dynamics

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\* Corresponding Author: Email: m\_haghjoo@sbu.ac.ir

## 1. Introduction

In recent decades, inspired by human movement and utilization of the characteristic dynamic stability of biped robots, researchers have aimed to reduce complexities and create more natural movements in these robots. During this period, dynamic-based control methods, such as proportional-derivative feedback control, were used to achieve asymptotic stability in active biped robots. This method, a type of time-variant controller, operates based on timing error. Consequently, many researchers have tried to replace the proportional-derivative feedback method with a more suitable approach for controlling biped robots. Notably, Grizzle et al. [1, 2] introduced the concept of "Hybrid Zero Dynamics," designing controllers based on time-invariant methods for the movement of biped robots. These methods allowed robots to move more naturally and consume less energy, making them more similar to human movement. Most research has concentrated on presenting and examining the control methods mentioned above for individual robots. According to the authors' findings, a comprehensive and fundamental parametric study on adjusting control coefficients in dynamic-based control methods to improve the walking of active biped robots still needs to be conducted. This research aims to conduct a parametric study of model-based dynamic methods to enhance the walking of active biped robots. Two widely used linearization feedback methods, "Hybrid Zero Dynamics" and "Proportional-Derivative Feedback," are utilized as a case study. For quantitative comparison, a simulation of controlling a three-link active biped robot is performed using MATLAB software. The design details of the controller using both methods are examined and discussed. In addition, the effects of different control gains in the proportional-derivative feedback method are examined, and their results are compared. Therefore, the key novelties of this research include 1) A parametric study of two traditional model-based dynamic methods for controlling the walking of biped robots, namely Hybrid Zero Dynamics and Proportional-Derivative Feedback. 2) An analysis of sensitivity and quantitative comparison of the effects of control parameters on stability indices and walking improvement in active biped robots, such as the timing of left and right steps, average speed, control effort, and more.

## 2. Dynamic Modeling

The model used in this research is a biped robot, as depicted in Figure 1, similar to the one referenced in [1]. Each walking step of the robot consists of two phases: 1) the continuous phase and 2) the impact phase. In the

continuous phase, it is assumed that the friction between the ground and the foot in contact is sufficiently high to prevent slipping, and the reaction force between the stance leg and the ground remains consistently positive (upward) [3]. It is important to note that the Lagrangian method will be used to obtain the dynamic equations governing this robot in both phases [3].

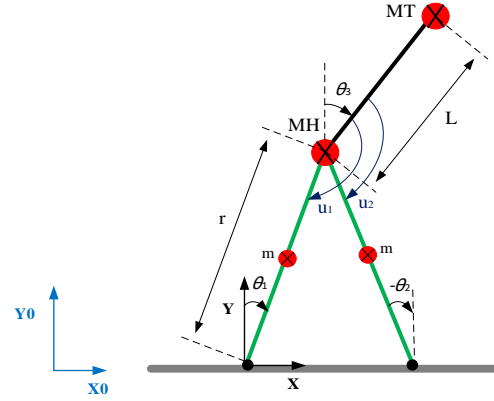


Figure 1. Model of a planar three-link biped robot

## 3. Asymptotically Stable Walking Control

Since the robot under study has three degrees of freedom, two virtual geometric constraints are required. One of these constraints is the hip angle, which must follow a desired value. In addition, for symmetrical movement, the angle between the stance and the swing legs must be equal and opposite. Therefore, the system output for this robot is defined as follows [4]:

(1)

The objective of the designed controller is to maintain the output at zero. The robot will achieve a stable periodic gait cycle if this vector converges to zero. To determine the required control law, the derivative of the system output is taken to obtain the control signal. The control signal can then be calculated as follows [4]:

(2)

The extracted control signal is utilized in both the proportional-derivative feedback control method and the zero-dynamics combination method. However, the difference between these two methods lies in how the dynamic system's output and the control signal's ideal function are derived. The ideal functions for the control signal in both methods are presented in Equations (3) and [4].

#### 4. Results and Discussion

Figure 2 shows the phase diagram and limit cycles formed for the motion of each case. In the HZD method, the timing of the left and right steps in the resulting limit cycle is relatively symmetrical, and the walking speed during the steps is uniform. In contrast, in the PD method, especially at lower gains, the timing of left and right steps is asymmetrical, and the movement is significantly uneven. However, as the control gain of the PD method increases, this irregularity decreases, and the movement speed becomes more uniform, similar to the HZD method. Furthermore, as the PD control gain increases, the duration of each step continuously decreases compared to before.

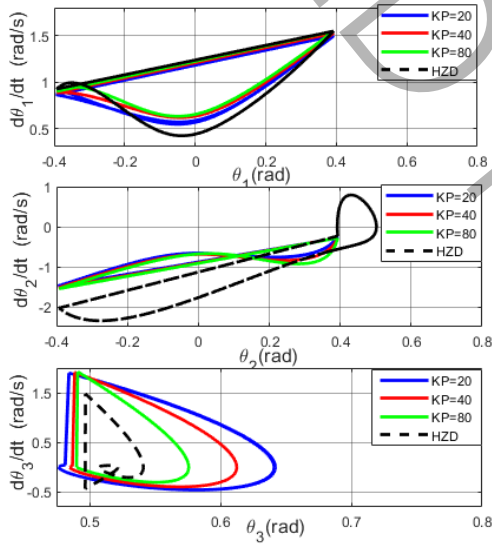


Figure 2. Phase diagrams of limit cycles

To further examine the performance of the controllers in terms of input torque to the system, the control effort during one step is compared in Table 1. For all methods, the average control effort and maximum torque of the supporting leg (the torque between the hip and the stance leg) are generally more significant than that of the swing leg (the torque between the hip and the swing leg). On the other hand, the maximum torques of the legs in the PD method are lower than in the HZD method for a low gain of  $K_P = 20$ . However, as the PD gain increases, the maximum torque of the stance leg becomes more extensive than that of the HZD method. However, the maximum torque of the swing leg remains almost unchanged with varying PD gains.

Table 1. Comparison of the average control effort and the average speed of the robot in two control methods

parameter(unit)	HZD	$K_P = 80$	$K_P = 20$
Control effort 1 ( $N.m.s$ )	0.047	0.106	0.213
Control effort 2 ( $N.m.s$ )	0.003	0.009	0.021
Maximum of torque 1 ( $N.m$ )	51.59	99.76	39.16
Maximum of torque 2 ( $N.m$ )	45.6	18.81	19.45

#### 5. Conclusions

In this article, a parametric study of model-based dynamic control methods was conducted to enhance the locomotion of underactuated biped robots. As a case study, two methods—Hybrid Zero Dynamics (HZD) and Proportional-Derivative Feedback (PD)—were applied and compared. The comparison of the results indicates that the HZD method produces more symmetric movement with smoother speed than the PD method, requiring less control effort. Furthermore, because the HZD method is time-invariant, it may exhibit lower sensitivity to potential errors that could arise during practical control. However, implementing HZD is more challenging due to the complexity of the relationships and configurations involved, and it is computationally more expensive than the PD method. As the control gain in the PD method increases, the results tend to converge closer to those of the HZD method, leading to a reduction in control effort.

#### 6. References

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