Elastostatic Modeling and Optimal Design of Rhombic Compliant Mechanism

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ABSTRACT

Compliant mechanisms are designed and used for precise positioning and amplification of piezoelectric actuators Due to their integrated structure. Modeling the kinematic behavior of these mechanisms has challenges due to their continuous structure and elastic deformation. This article presents a structural matrix-based method called the elastostatic method for static modeling of compliant mechanisms. The innovation of elastostatic modeling reduces calculations by approximating rotation and small displacement. The main goal of this research is to design and optimize the rhombus flexible mechanism using elastostatic modeling. This mechanism is optimized in such a way that, in addition to positioning, it has high magnification and low input stiffness. The rhombus mechanism has an integrated and simple structure and is used for micron positioning and piezoelectric actuator amplification. In this research, the rhombus mechanism has been modeled using the elastostatic method, and its dimensions have been optimized according to the parameters of the mechanism; For this purpose, it is necessary to check the modeling error. The modeling error is compared with simulation in finite element software and experimental results. The results show that the modeling used to design the rhombus mechanism have a 1.5% error compared to experimental results.

KEYWORDS

elastostatic modeling, compliant mechanism, rhombic mechanism, optimization, mechanism design

1. Introduction

Compliant mechanisms, which transmit motion through the elastic deformation of their components, have gained significant attention due to their high precision and elimination of friction when compared to traditional mechanisms [1]. These advantages make them particularly suitable for applications requiring micro-positioning and actuator amplification [2]. The absence of assembly and lubrication needs further enhances their appeal [3].

The use of compliant mechanisms in various fields, such as micro-electromechanical systems (MEMS), biomedical devices [4], and precision engineering , underscores their importance [5]. The ability to design mechanisms that can achieve significant displacement amplification while maintaining low input stiffness is crucial for applications involving piezoelectric actuators. Piezoelectric materials, which convert electrical energy into mechanical displacement, often require amplification mechanisms to achieve the desired range of motion [6].

This study focuses on the design and optimization of a rhombic compliant mechanism using elastostatic modeling. The objective is to achieve high[7] magnification and low input stiffness, making the mechanism suitable for precise positioning and amplification applications. Traditional methods often involve complex calculations and extensive computational resources, but elastostatic modeling offers a more efficient alternative by approximating small displacements and rotations, thus simplifying the overall process.

2. Elastostatic modeling

The methodology focuses on the elastostatic modeling of the rhombic compliant mechanism. This modeling approach simplifies the calculation process by approximating small displacements and rotations, thus reducing computational complexity. The rhombic mechanism is selected due to its integrated structure, which is advantageous for micron-level positioning and the amplification of piezoelectric actuators. The elastostatic model is constructed by defining the relationship between forces and displacements within the mechanism [8]. The stiffness matrix represents the mechanism's stiffness and deformation characteristics. The process begins with formulating the potential energy of the system, which comprises the elastic strain energy stored in the compliant links as mentioned in Eq (1) .

$$V = \frac{1}{2}\begin{bmatrix} \boldsymbol{U}_i^T & \boldsymbol{U}_j^T \end{bmatrix} \boldsymbol{Q}^T \begin{bmatrix} \boldsymbol{K}_{ii} & \boldsymbol{K}_{ij} \\ \boldsymbol{K}_{ji} & \boldsymbol{K}_{jj} \end{bmatrix} \boldsymbol{Q} \begin{bmatrix} \boldsymbol{U}_i \\ \boldsymbol{U}_j \end{bmatrix} \qquad (1)$$

With the evaluation of the strain energy for all flexible links and summing them up the total strain of the system is obtainable. With obtaining the total strain energy the equivalent stiffness matrix equation is obtainable as shown in Eq (2)

$$\boldsymbol{C}_i = \frac{\partial^2 V_i}{\partial \boldsymbol{F}^2} \rightarrow \boldsymbol{K}_i = \boldsymbol{C}_i^{-1} \qquad (2)$$

As a result, the motion of the mechanism could be calculated relative to the input force.

For small displacements and rotations, the behavior of the mechanism can be approximated linearly. This simplification allows for efficient calculations while maintaining accuracy. The small displacement assumption is particularly valid for compliant mechanisms, where deformations are typically within a small range.

The elasostatic method have been implemented on rhombic mechanism shown in Figure 1. Then the dimensions of the rhombic mechanism are optimized to ensure high amplification and low input stiffness. This involves determining the optimal length *L*, width *w*, and thickness *t* of the compliant segments. The optimization process aims to maximize the displacement amplification ratio while adhering to design constraints.

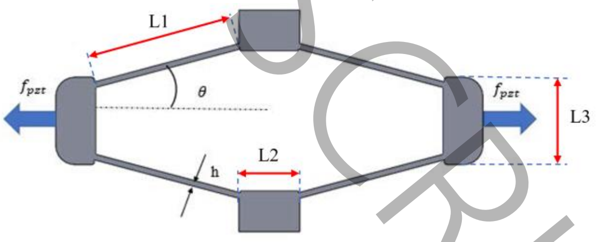

Figure 1. rhombic mechanism

3. Experimental Validation

The accuracy of the elastostatic model is validated by comparing the results with finite element simulations and experimental data. The experimental setup involves applying known forces to the mechanism and measuring the resulting displacements. The comparison of the elastostatic model with finite element analysis (FEA) results shows a high degree of correlation, validating the assumptions and simplifications made during the modeling process. The experimental validation further confirms the model's accuracy, with minor discrepancies attributed to manufacturing tolerances and material properties.

4. Results and Discussion

The results of the elastostatic modeling of the rhombic mechanism were compared against finite element simulations and experimental data. The accuracy of the model was confirmed, with an error margin of approximately 1.5% for displacement amplification ratio and 1% for output displacement. The optimization process focused on adjusting the geometric parameters of the rhombic mechanism to maximize its amplification and minimize input stiffness. The optimized dimensions resulted in significant improvements in the performance of the mechanism. The results indicate that the elastostatic model provides a reliable and accurate representation of the rhombic mechanism's performance. The optimized mechanism demonstrates high amplification and low input stiffness, making it suitable for precise positioning and amplification applications.

The comparison of the model with finite element analysis (FEA) results shows a high degree of correlation, validating the assumptions and simplifications made during the modeling process. The experimental validation further confirms the model's accuracy, with minor discrepancies attributed to manufacturing tolerances and material properties

5. Conclusions

This research successfully demonstrates the application of elastostatic modeling in the design and optimization of a rhombic compliant mechanism. The method effectively reduces computational complexity and provides accurate results, with an error margin of around 1.5% compared to experimental data. The optimized mechanism shows significant improvements in magnification and input stiffness, making it suitable for precise positioning and amplification in various applications.

The study highlights the potential of elastostatic modeling as a valuable tool for the design and optimization of compliant mechanisms. Future work may explore further enhancements in modeling accuracy and the application of this approach to other types of compliant mechanisms. Additionally, the integration of this method with other optimization techniques could yield even better performance and broader applicability.

The findings also suggest that the elastostatic modeling approach can be extended to more complex mechanisms and systems, providing a robust framework for the analysis and design of compliant structures. The continued development of this method could lead to significant advancements in the fields of precision engineering, MEMS, and beyond.

6. References

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