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Static Analysis of Bending, Stability, and Dynamic Analysis of Functionally Graded Plates by a Four-Variable Theory

F. Kaviani [1]& H. R. Mirdamadi[2*]

1- M.Sc. Student, Department of Mechanical Engineering, Isfahan University of Technology, Isfahan, Iran
2- Assistant professor, Department of Mechanical Engineering, Isfahan University of Technology, Isfahan, Iran

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ABSTRACT

In this article, static analysis of bending, elastic stability, and free vibration analysis of functionally graded plates (FGP) are investigated using a four-variable theory. In this four-variable theory, hyperbolic sine distribution is used for satisfying boundary conditions of out-of-plane shear stresses of zero value, in the neighborhood of upper and lower surfaces of plate. One of the mechanical characteristics of FGM material is continuous variations of properties along the thickness, with a power law distribution, which is a function of volume ratio of different constituent parts of FGM plate. The purpose of this article is acquiring more exact analytical results than those of simple form of four-variable plate theory, i.e., refined plate theory (RPT). Furthermore, for parametric study, influential parameters on the analysis of FGM plate are investigated. The plate equations of motion are derived by extended Hamilton's variational principle. Analytical results are developed based on classical method of Navier and simply-supported conditions on all four edges. Numerical results are analyzed for different power distributions of mechanical properties along the thickness and different plate length to thickness ratios. The results, obtained from this theory, are compared with those of different variants of RPT theories.

KEYWORDS

Functionally Graded Plate (FGP), Refined Plate Theory (RPT), Four-Variable Theory, Out- of- Plane Shear Stresses, Navier's Solution.

* Corresponding Author, Email: hrmirdamadi@cc.iut.ac.ir

1- Introduction

Shimpi [1] introduced a refined theory of plates for both reaching more accurate results and preserving the simple form of classical plate theory. He used two sets of variables, one of those was variationally consistent and the other, variationally inconsistent. In the set of variationally consistent variables, the undetermined variables of the system generate the equations of the motion of plate independently, such that these equations would resemble the equations by Reissner, while one of the equations would become quite similar to those of classical plate theory. By using equations of equilibrium, in that article, a new constraint is developed among transversal displacement variables, for flexural and shear components, extracted from a set of variationally inconsistent variables. This new relation causes a reduction in the number of system variables. In [2], by ignoring the effect of extensional component in the transversal displacement, in the proposed theory of Shimpi, a four-variable theory is derived.

For functionally graded plates, the refined plate theory could cause better accuracy for numerical computations as well as possessing an inherently simple theory. In this article, we use the four-variable theory by using a new hyperbolic distribution function for shear strain. Therefore, we employ these merits accompanying with a hyperbolic sine function for thickness-through shear strain distribution. Selecting this distribution for shear strains, together with having a simple theory for capturing this type of distribution, we could reach numerical results with more accuracy.

2- Formulation

In the four-variable theory, the displacement components are as follows,

$$u(x,y,z,t)=u_a(x,y,t)-z\frac{\partial w_b}{\partial x}-f(z)\frac{\partial w_s}{\partial x} \tag{1}$$

$$v(x,y,z,t)=v_a(x,y,t)-z\frac{\partial w_b}{\partial y}-f(z)\frac{\partial w_s}{\partial y}$$

$$w(x,y,z,t)=w_b(x,y,t)+w_s(x,y,t)$$

$$f(z)=\frac{h\sinh(10\frac{z}{h})}{10\cosh(5)}-\frac{h}{100}$$

The mechanical properties of the plate are assumed to have thickness-through distributions as,

$$E(z)=(E_c-E_m)(\frac{4z^2}{h^2})^P+E_m \tag{2}$$

$$\rho(z)=(\rho_c-\rho_m)(\frac{4z^2}{h^2})^P+\rho_m$$

By using the above-mentioned distributions, the equations of motion in terms of displacements are as follows,

$$A_{11}\frac{\partial^2 u_a}{\partial x^2}+A_{66}\frac{\partial^2 u_a}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 u_a}{\partial x\partial y}=\rho_0\ddot{u}_a \tag{3}$$

$$A_{22}\frac{\partial^2 v_a}{\partial y^2}+A_{66}\frac{\partial^2 u_a}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 u_a}{\partial x\partial y}=\rho_0\ddot{v}_a$$

$$-[D_{11}\frac{\partial^4 w_b}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w_b}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w_b}{\partial y^4}]$$
$$-[D_{11}^s\frac{\partial^4 w_s}{\partial x^4}+2(D_{12}^s+2D_{66}^s)\frac{\partial^4 w_s}{\partial x^2\partial y^2}+D_{22}^s\frac{\partial^4 w_s}{\partial y^4}]+q+\aleph(w)$$
$$=\rho_0(\ddot{w}_b+\ddot{w}_s)-\rho_2(\frac{\partial^2\ddot{w}_b}{\partial x^2}+\frac{\partial^2\ddot{w}_b}{\partial y^2})$$

$$-[D_{11}^s\frac{\partial^4 w_b}{\partial x^4}+2(D_{12}^s+2D_{66}^s)\frac{\partial^4 w_b}{\partial x^2\partial y^2}+D_{22}^s\frac{\partial^4 w_b}{\partial y^4}]$$
$$-[H_{11}^s\frac{\partial^4 w_s}{\partial x^4}+2(H_{12}^s+2H_{66}^s)\frac{\partial^4 w_s}{\partial x^2\partial y^2}+H_{22}^s\frac{\partial^4 w_s}{\partial y^4}+A_{44}^s\frac{\partial^2 w_s}{\partial y^2}$$
$$+A_{55}^s\frac{\partial^2 w_s}{\partial x^2}]+q+\aleph(w)=+\rho_0(\ddot{w}_b+\ddot{w}_s)-\rho_0^2(\frac{\partial^2\ddot{w}_s}{\partial x^2}+\frac{\partial^2\ddot{w}_s}{\partial y^2})$$

Based on the selected simply-supported boundary conditions, the solution for this boundary-value problem is followed by selecting exact interpolation functions.

Figure 1: Coupled effects of modulus ratio of ceramic and metal constituents of FGM plate and the exponent of distribution law of FGM properties, on the plate out-of-plane deformations.

3- Numerical results

The transverse deflections of a plate are dependent on the coupled effects such as modulus ratio and exponent of distribution law for FGM properties, P. This coupling is such that with an increase in the modulus ratio, the effect of P on lateral displacements shows an increase. Moreover, with increasing P, the effect of modulus ratio on the lateral displacements demonstrates an increase as well.

With simultaneous changes in the modulus ratio and exponent of FGM law, different changes could occur in the plate fundamental frequency, which is depicted in Fig. 2.

Figure 2: Coupled effects of modulus ratio of ceramic and metal constituents of FGM plate and the exponent of distribution law of FGM properties, on the plate fundamental frequency.

The simultaneous influence of elastic modulus ratio of the ceramic part of plate to the metal part and the exponent of FGM property law, i.e., P, could be observed in Fig. 3.

Figure 3: Coupled effects of modulus ratio of ceramic and metal constituents of FGM plate and the exponent of distribution law of FGM properties, on the plate critical buckling loads.

4- Conclusions

In this article, a modified form of classical four-variable theory was developed for FGM systems. This theory represents transversal shear strains by using hyperbolic distribution functions. The current theory could satisfy the zero value boundary conditions for shear stresses adjacent to upper and lower skins of the plate. The bending and free vibration exact analysis results of this theory for FGM systems were the pivotal subject of this article. One of characters of this theory would be the use of RPT together with its simple form of equations, as well as its higher precision. Therefore, the simple form of this theory could lead to reduction of the complexity in the solution process as well as simplicity in gaining more sense to the physics of governing equations. Moreover, the investigation of plates having lower aspect ratios of length to thickness (thick plates) would have special importance in the stress analysis of structural plates. The current theory offers different results for thick plates and this point could be a motivation for more development of this theory. Of characters of this investigation, we could address the study of effects of different parameters of FGM plates on their static and dynamic behavior. For instance, increasing the exponent of the power law distribution of material properties, given a constant modulus ratio (i.e., Ec/Em), would cause an increase the out-of-plane displacements of the plate while it could cause decreases in both its critical compressive load and fundamental frequency.

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