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Exponential Basis Functions in Solution of Time – Dependent Heat Equation in Axially Layered Materials

B. Movahedian Attar[1], B. Boroomand[2*]

1- Ph.D Student, Department of Civil Engineering, Isfahan University of Technology, Iran
2- Professor, Department of Civil Engineering, Isfahan University of Technology, Iran

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ABSTRACT

In this paper, we present a novel method based on using Exponential Basis Functions (EBFs) to solve the heat conduction problem in axially layered materials. In the first step, we have considered each layer of material as a separate element. Then the solution in each element was approximated by a summation of EBFs satisfying the differential equation of the transient heat conduction problem. The unknown coefficients of the series solution were related to initial condition and Dirichlet side conditions of each layer employing a discrete transformation technique. Finally, the general solution of material was completed by satisfying the continuity conditions between adjacent layers in a manner similar to the conventional finite element method. In this hybrid method, a collocation scheme was used for satisfying the time dependent boundary conditions as well as the initial conditions. The capability of the presented technique was investigated in the solution of some benchmark problems.

KEYWORDS

Transient Heat Conduction, Layered Materials, Meshless Method, Exponential Basis Functions, Discrete Transformation.

* Corresponding Author, Email: boromand@cc.iut.ac.ir

1- INTRODUCTION

Many problems in physics and engineering require knowing the accurate distribution of transient temperature within the multi layered material. A survey in the literature shows that several analytical methods have been proposed to solve the heat conduction problem in layered materials, up to now [1]. However, in some cases, there is not any analytical solution and thus the development of accurate numerical methods is a necessity. In this regard, a variety of numerical methods such as finite difference method (FDM), finite element method (FEM) or boundary element method (BEM) have been developed. The method of Fundamental solutions (MFS) is one of the numerical meshless methods capable of solving some classes of time dependent differential equations. Recently, the MFS has been employed to solve the transient heat conduction in the layered materials [2].

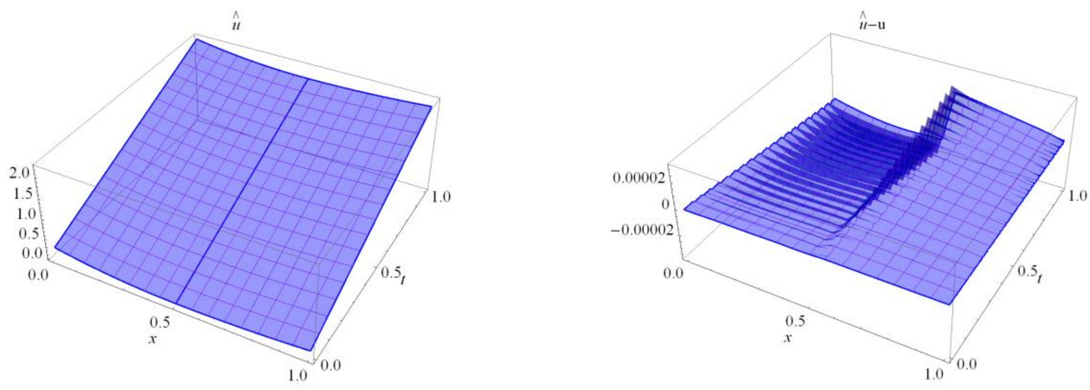
In this paper, we extend the meshless method introduced in [3-5] to solve the transient heat conduction problem in the layered materials. Using the exponential basis functions as the basis of approximate solution is the main idea in the presented method. A discrete transformation technique is also employed for the simultaneous satisfaction of initial/boundary conditions of the problem in a collocation manner.

2- METHODOLOGY

There are several techniques for the solution of transient problems such as Laplace transformation or finite difference method. Here, we introduce a novel idea for the time dependent problems. Considering time as an axis, we treat the problems in a manner similar to the one introduced in [3] while acknowledging the fact that the problem is of the initial value type in this case. We present a hybrid method to determine the temperature field in the whole of domain. In the first step, each layer of the material was considered to be a separate element. Then, a series of the exponential basis functions (EBFs) were used to construct the solution in each layer. A discrete transformation technique, proposed in our previous works [3], has been employed to relate the unknown coefficients of each layer to the initial condition and Dirichlet side conditions. Moreover, the general solution of the considered material was obtained through satisfying the continuity condition between the adjacent elements in a manner similar to FEM. Likewise, we introduce a time marching procedure with the aid of the proposed method. To this end, we choose a small time interval and repeat the procedure in a step by step manner while using the information obtained at the end of time interval as the initial values for the next step.

3- NUMERICAL RESULTS

The capability of the presented method was investigated in the solution of three sample problems. The obtained results indicate that the technique accurately estimate the response , especially in comparison to the other available numerical methods (See Ref [2] for the results of MFS). The numerical temperature field and its error distribution are shown in Figure 1 for a two layered material.



(a) (b)

Figure 1. The numerical temperature field (a) and its error distribution (a) in a two layerd material

The ability of the method to solve the problems in materials with more layers was also tested. (See Figure 2 for the temperature distribution in a five layered material due to the thermal shock on the right side).

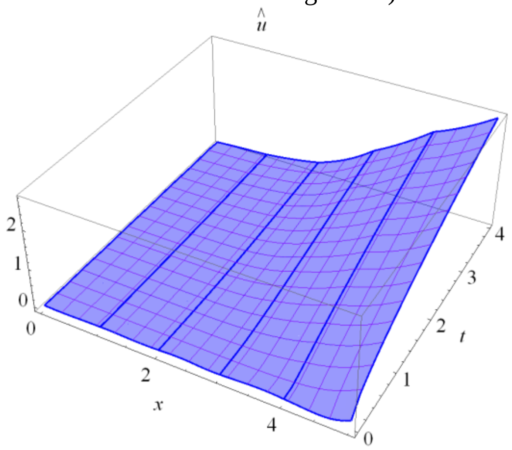


Figure 2. The numerical temperature field in a five layerd material due to thermal shock

4- CONCLUSIONS

We have presented a hybrid method based on using exponential basis functions (EBFs) for the solution of transient heat conduction problems in layered materials. The EBFs are restricted to satisfy the time dependent differential equation. The interface conditions between adjacent layers have been also satisfied in a manner similar to conventional finite element method. The capability of the presented method is shown in the solution of some sample problems.

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