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Analysis of the Nonlinear Behavior of the Free Vibration of a Cantilever Beam with a Fatigue Crack Using Lindstedt- Poincare's Method

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ABSTRACT

Previous investigations indicate that using the open crack model for vibration analysis of cracked structures may lead to incorrect results. Such a simple model can only be used as a rough approximation for predicting the dynamic behavior of the cracked structures. Therefore, in order to predict the nonlinear dynamic behaviour of the structures with a fatigue crack more accurately, one has to consider the nonlinearity of the crack. In this paper, the nonlinear behavior of the free vibration of a cantilever beam with a Fatigue Crack is investigated. To this end, first, the lateral vibration of the cracked beam in its first mode is modeled as an SDOF system with an equivalent mass and stiffness. Then, a new model is introduced for the bilinear stiffness of the beam with a breathing crack. Using this model, the governing differential equation of motion is converted to the standard form that can be analyzed by Lindstedt- Poincare's method. The results show that the response is composed of two parts. The main part is the response of a system with the mean equivalent stiffness of the systems corresponding to the closed crack and the open crack cases. The second part is composed of the first and second order correction terms, which reflects the effect of opening and closing of the crack on the vibration response. In fact, the correction terms consist of the higher harmonic components of the spectrum. The results have been validated by the experimental tests.

KEYWORDS

Cracked Beam, Breathing Crack, Bilinear Crack Model, Lindstedt Poincare Method.

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1- INTRODUCTION

In the most studies in the area of fault detection based on the vibration analysis, the linear models are used. However, some researchers showed that the local stiffness at the crack location changes during the structure vibration. Therefore, the vibration behavior of the cracked structure cannot be predicted through using the linear models, properly. Elber [1] investigated the effect of opening/closing the crack on the vibration behavior of a cracked cantilever beam. Gudmundson [2] studied the crack closure effect on an edge-cracked beam. He found that the reduction in natural frequencies due to the closing crack is much smaller than the open crack. Friswell and Penny [3] have modeled the nonlinear behavior of a cantilever beam with a breathing crack at its first vibration mode by using an SDOF oscillator with a bilinear stiffness.

Recently, Peng et al. [4] investigated the nonlinear behavior of a cracked beam using the concept of nonlinear frequency response function. They showed that the response of the system has a high sensitivity to the fatigue crack when it is excited at a proper frequency.

In the present study, by introducing a new bilinear stiffness model for a breathing crack, the lateral vibration of a cracked cantilever beam is investigated. The governing differential equation is analyzed by the Lindstedt Poincare's method. The theoretical results are verified by the experiments.

2- MODELING OF CRACKED CANTILEVER BEAM

In this study, free vibration of a cantilever beam with a fatigue crack at first vibration mode is modeled as an SDOF system. The equivalent mass and the stiffness of the cracked beam are obtained. The bilinear stiffness is modeled as:

$$k(u)=\begin{cases}k_{close} & u>0\\ k_{open} & u<0\end{cases} \tag{1}$$

where k_{open} and k_{close} are the equivalent stiffness of system at the fully open and the fully closed states of the crack, respectively. The equivalent stiffness of the intact beam may be written as [2]:

$$k_c=\frac{1}{C_c}=\int_0^L EI\phi''^2(x)dx \tag{2}$$

where C_c is the compliance of the intact beam, E is the Young modulus, I is the cross section moment of inertia, and L is the beam length. Clough and Penzien [5] suggested the first vibration mode of the cantilever beam as the following:

$$\psi(x)=1-\cos\left(\frac{\pi x}{2L}\right) \tag{3}$$

In the present study, in order to improve the model, instead of Eq. (3), the first vibration mode of the cantilever Euler-Bernoulli beam is used:

$$\phi(x)=\cos\lambda x-\cosh\lambda x-\frac{(\cos\lambda L+\cosh\lambda L)}{(\sin\lambda L+\sinh\lambda L)}(\sin\lambda x-\sinh\lambda x) \tag{4}$$

In which, at the first vibration mode, $\lambda L=1.875$.

The equivalent stiffness of the beam with a fully open crack may be written as:

$$k_o=\frac{1}{C_o} \tag{5}$$

In the above equation, $C_o=C+\Delta C$ and ΔC is the changes in compliance due to the crack. Dimarogonas and Paipetis [6] obtained ΔC in terms of the crack depth, a, as:

$$\Delta C=w\frac{\partial^2}{\partial P_i^2}\int_0^a Jda=\frac{72l_c^2\pi(1-\nu^2)}{Ewb^4}\varphi \tag{6}$$

where w and b are the width and the height of the beam cross sectional area, respectively. J is the strain energy release rate, ν is the Poisson ratio, and φ is a function of the relative crack depth and is:

$$\varphi=19.60\frac{a^{10}}{b^8}-40.69\frac{a^9}{b^7}+47.04\frac{a^8}{b^6}-32.99\frac{a^7}{b^5}+20.30\frac{a^6}{b^4}-9.98\frac{a^5}{b^3}+4.60\frac{a^4}{b^2}-1.05\frac{a^3}{b}+0.63a^2 \tag{7}$$

According to above explanations, the governing equation of motion for the cracked beam which is modeled as an SDOF system may be expressed as:

$$m\ddot{u}+k(u)u=0 \tag{8}$$

where u is the displacement of the beam equivalent mass.

3- NEW BILINEAR MODEL OF THE CRACKED BEAM

By defining the following parameters:

$$k=\frac{k_{open}+k_{close}}{2},\quad \varepsilon=\frac{k_{close}-k_{open}}{k_{close}+k_{open}} \tag{9}$$

the new model for the bilinear equivalent stiffness is introduced as:

$$\begin{cases}k_{close}=k(1+\varepsilon)\\ k_{open}=k(1-\varepsilon)\end{cases} \tag{10}$$

Using Eqs. (9) and (10), one can express the equivalent stiffness as:

$$k(u)=ku+\varepsilon k|u| \tag{11}$$

By performing some mathematical manipulations, the governing equation of motion can be written as follows:

$$\ddot{u}+\omega_0^2u=-\varepsilon\omega_0^2|u| \tag{12}$$

where ω_0 is the linear natural frequency.

4- ANALYZING THE EQUATION BY THE LINDSTEDT POINCARE' METHOD

Based on the Lindstedt Poincare's method, the solution of equation (12) can be assumed as:

$$\begin{cases}u=u_0+\varepsilon u_1+\varepsilon^2u_2+...\\ \omega=\omega_0+\varepsilon\omega_1+\varepsilon^2\omega_2+...\end{cases} \tag{13}$$

By defining $\tau=\omega t$, substituting Eq. (13) into Eq. (12), and equating the coefficients of the same powers of ε, the following equations are obtained:

$$\varepsilon^0: \quad \omega_0^2\left(\frac{d^2u_0}{d\tau^2}+u_0\right)=0 \tag{14}$$

$$\varepsilon^1: \quad \omega_0^2\left(\frac{d^2u_1}{d\tau^2}+u_1\right)=-2\omega_0\omega_1\frac{d^2u_0}{d\tau^2}-\omega_0^2|u_0| \tag{15}$$

$$\varepsilon^2: \quad \omega_0^2\left(\frac{d^2u_2}{d\tau^2}+u_2\right)=-2\omega_0\omega_1\frac{d^2u_1}{d\tau^2} -\left(\omega_1^2+2\omega_0\omega_2\right)\frac{d^2u_0}{d\tau^2}-\omega_0^2\frac{|u_0+\varepsilon u_1|}{u_0+\varepsilon u_1}u_1 \tag{16}$$

Assuming pure displacement for an initial condition, the perturbation solution becomes:

$$u(\tau)=u_0(\tau)+\varepsilon u_1(\tau)+\varepsilon^2u_2(\tau) \tag{17}$$

where

$$u_0=A\cos\tau$$

$$u_1=-A\left\{\frac{2}{\pi}+\sum_{n=1}^{\infty}\frac{4}{\pi}\left\{\frac{1}{(1-4n^2)^2}(-1)^n\right\}\cos 2n\tau\right\} \tag{18}$$

$$u_2=b_0+\sum_{m=2}^{\infty}\left(\frac{b_m}{1-m^2}\right)\cos m\tau$$

Moreover, the natural frequency of the cracked beam is:

$$\omega=\omega_0+\varepsilon^2\omega_2 \tag{19}$$

where

$$\omega_2=\frac{\omega_0 b_1}{2A} \tag{20}$$

5- RESULTS AND DISCUSSION

The geometrical and the mechanical properties of the cantilever beam containing a breathing crack are chosen as: L = 450 mm, w = 3.9 mm, b = 6.4 mm, E = 200 GPa and ρ =7860 kg/m^3.

To validate the theoretical results, some experimental tests are performed on the cracked beam. To this end, a fatigue crack at a relative location of $\beta=0.8$ and a relative depth of $\alpha=0.38$, was created on the beam by a servohydraulic universal dynamic test machine (Zwick/Roell Amsler HA250). The dynamic response of the beam was captured using an accelerometer (B&K 4507) and a signal analyzer (B&K 3109).

Moreover, the perturbation solution is validated by the numerical technique (RK4). The results show that considering only one correction term in the perturbation solution, the response closely coincides with that obtained by the numerical technique.

The theoretical results obtained through the Lindstedt Poincare's method revealed that the vibration response of the cracked beam is influenced by the crack depth. In fact, for a given crack location, as the crack depth increases, the effect of the correction term, the second term on the right hand side of Eq. (13), which is reflecting the influence of the crack on the vibration response, becomes considerable (Fig. 1).

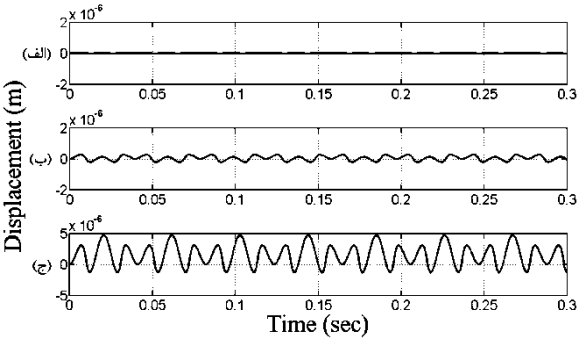

Fig. 1 The contribution of the correction term in vibration response of the cracked beam for the relative crack location of $\beta=0.6$ and the crack relative depth of (a) $\alpha=0.3$, (b) $\alpha=0.5$ and $\alpha=0.7$

6- CONCLUSION

In this research, a new bilinear stiffness model is proposed for a beam with a fatigue crack. The governing equation of motion is solved by the Lindstedt Poincare's method. The main results are:

- The relationship among the harmonic components and the crack parameters of the beam with a fatigue crack is appeared explicitly in the vibration response which is obtained through the perturbation method.
- Although the cracked beam vibrates mainly at its fundamental frequency, weak harmonic components are also present in the response due to the nonlinearity of the crack. Such a behavior is observed from the experiments.
- Comparing the cracked beam fundamental frequency obtained through using the two crack models, i.e. the presented model for the breathing crack and the open crack model, and that obtained by the experiment shows that the one obtained through applying the presented model are more accurate than the open crack model.

7- REFERENCES

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