AmirKabir University of Technology
(Tehran Polytechnic)

AmirKabir Jounrnal of Science & Research
(Mechanical Engineering) (ASJR-ME)

Vol. 47, No. 2, Winter 2015, pp. 17-20

Behavior of FGM Spherical Vessels Under Internal Pressure and Temperature Difference

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(Received 2 June, 2012, Accepted 12 May, 2015)

ABSTRACT

In this paper, FGMs are used as non-uniform materials in high temperature environments. Different industries use them in thin and thick walled spherical pressure vessels. Based on governing equations, differential equation of stresses is obtained in plastic state that can be widely used in the study of reservoirs behavior in elasto-plastic state. The study has discussed on temperature distribution and stress - strain relationships in the tanks under internal pressure and temperature difference. Properties of these materials are considered as variable parameters function of radius. In this work, effects of these parameters have been investigated on yielding, yield temperatures and stress changes in thickness of the vessels. Furthermore, it is shown that vessels structure can be optimized by choosing appropriate parameters.

KEYWORDS:

Linear kinematic hardening, FGM, Prager's model, Spherical vessel, Yield surface

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1- Introduction

Functionally graded materials (FGM) are heterogeneous materials in which the elastoplastic and thermal properties change from one surface to the other, gradually and continuously. Classical method of analysis combines the equilibrium and compatibility equations with the stress–strain relations to derive governing equations in terms of the stress components.

Tutuncu and Ozturk [1] presented the closed-form solutions for stresses and displacements in functionally graded cylindrical and spherical vessels solely subjected to internal pressure. An analytical method is developed to obtain solution for two-dimensional steady state thermomechanical stresses in a hollow thick sphere made of functionally graded material by Poultangari et al. [2]. We can see the different results and condition in spherical vessels by several studies [3-8]

In the present work, FGM has been used in thin and thick walled spherical pressure vessels, which are extensively used in industry. Based on the governing equations, differential equation of stress is obtained in plastic state that can be widely used in the study of reservoirs behavior in elasto-plastic state.

2- Governing Equations

Consider a thick spherical vessel of inside radius, a, and outside radius, b, made of FGM. It is assumed that the mechanical and thermal loads and their associated boundary conditions are such that the stress field is a function of radius because of the symmetry. The following material parameters can be written as Eq.(1).

$$k(r)=k_0\left(\frac{r}{b}\right)^{m_1},\ E(r)=E_0\left(\frac{r}{b}\right)^{m_2}$$
$$\alpha(r)=\alpha_0\left(\frac{r}{b}\right)^{m_3},\ \sigma_y(r)=\sigma_0\left(\frac{r}{b}\right)^{m_4} \qquad (1)$$

We use the 1st law of thermodynamics and get the temperature distribution by thermal properties variation and boundary conditions.

$$\sigma_r=c_3r^{\lambda_1}+c_4r^{\lambda_2}+c_5r^{\lambda_1+l_1}$$
$$+c_6r^{\lambda_1+l_2}+c_7r^{\lambda_2+l_3}+c_8r^{\lambda_2+l_4} \qquad (2)$$

Also, the radial stress can be obtained by the Prager's model, von Mises criterion, associated flow rule [9], compatibility and equilibrium equations.

$$(Ar^{-m_2+1}+Br^{-m_6+1})\frac{d^2\sigma_r}{dr^2}+$$
$$(Cr^{-m_2}+Dr^{-m_6})\frac{d\sigma_r}{dr}+Gr^{-m_2-1}\sigma_r\pm \qquad (3)$$
$$Hr^{-m_6+m_4-1}+Ir^{m_3-m_1-2}+Jr^{m_3-1}=0$$

3- Numerical Results

According to the material parameters given in each row of Table 1 (without inside pressure) and Table.2 (without temperature difference), yield onset of the spherical vessels. In addition, the plastic deformation may commence simultaneously at both surfaces of the vessel according to Figure 1.

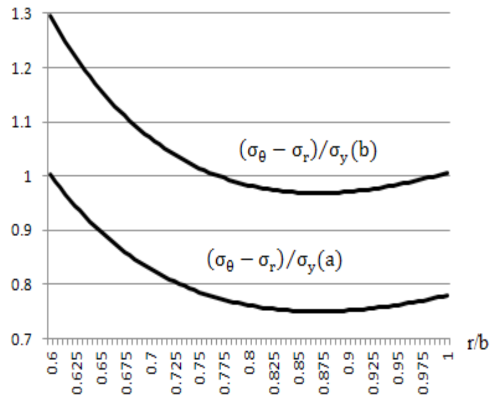

Figure 1. Variation of differences of the radial and tangential stresses in sphere versus the radius

Table 1. Result of spherical vessel without inside pressure with different parameters

Yield onset area	ΔTy (°C)	m_4	m_3	m_2	m_1
Inside	226	2	3	-1.1	3
Outside	155	-3	-1.5	-1.5	-2
Inside	31	3	-1.4	1.1	-2.1
Outside	243	-0.5	-0.5	0.5	-3
Inside	136	0.5	-0.7	0.5	-3

Table 2. Result of spherical vessel without temperature difference with different parameters

Yield onset area	P_y (MPa)	m_4	m_3	m_2	m_1
Inside	66.7	2	3	-1.1	3
Inside	11	3	-1.4	-1.5	-2.1
Inside	55	3	-1.4	1.1	-2.1
Inside	302	-0.5	-0.5	0.5	-3
Inside	181	0.5	-0.7	0.5	-3

For a profile, if we compare the radial stresses and tangential, since the role of tangential stresses is greater than the radius stresses, the temperature gradient has the opposite effect in creating stresses towards inner pressure loading. This causes the yield point to be reached at a later stage of the thermal loading. When the spherical vessel is only under internal pressure loading, difference of tensions at the inner radius are positive and tension and when the spherical vessel is under temperature gradient loading, difference of stresses at the inner radius are negative and compressive. For thin walled spherical vessel with specifications: $a/b=0.987$, $\Delta T=T_a-T_b=10°C$ and mechanical characteristics: $E_0=200$ GPa, $\alpha_0=11.7$ 1/°C, $\sigma_0=415$ MPa, $\nu=0.3$, $m_1=3$, $m_2=-1.1$, $m_3=3$ and $m_4=2$, we can see results in Figure 2.

One of the main results in this paper is obtaining a diagram by investigation of 4 parameters; pressure, m_4, T_a and T_b as shown in Figure 3.

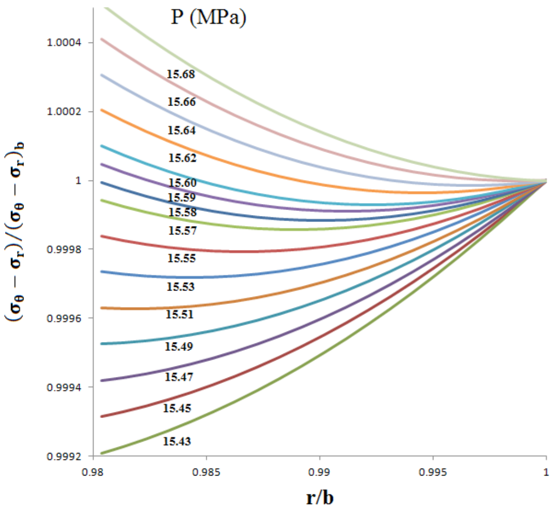

Figure 2. Variation of differences of the radial and tangential stresses in thin walled sphere versus the radius under different inside pressure

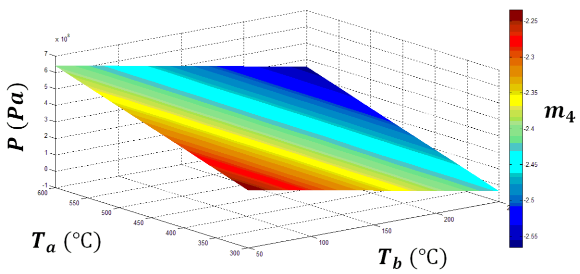

Figure 3. Variation of the spherical vessel parameters with specifications: $a/b=0.91$, $m_1=3$, $m_2=1.1$, $m_3=3$

4- Conclusion

In the present work, FGMs in spherical vessels are investigated. Properties of these materials are considered as variable parameters function of radius. Elastic-Plastic governing equations are obtained. The study has discussed on the temperature distribution and stress-strain relationships in the sphere under internal pressure and temperature difference. In this work, effects of these parameters have been studied on the yielding, the yield temperatures and stress changes in thickness of the spherical vessels. Furthermore it is shown that pressure vessels structure can be optimized by choosing appropriate parameters. In fact, by considering the four parameters, internal and external temperature, internal pressure and the yield stress, can be better analyzed on a FGM vessels.

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