AmirKabir University of Technology
(Tehran Polytechnic)

AmirKabir Jounrnal of Science & Research
Mechanical Engineering - ASJR-ME
Vol. 48, No. 1, Spring 2016, pp. 1-4

Numerical Solution of Liquid-Vapor Flow in Variable Cross-Section Ducts by Using Flux-Vector Splitting Method

A. Nouri[1*] , A. Shafiei G.[2]

1- Professor, Department of Mechanical Engineering, Sharif University of Technology, Tehran, Iran
2- Ph.D. Student, Department of Mechanical Engineering, Sharif University of Technology, Tehran, Iran

(Received 07 April, 2014, Accepted 6 September, 2015)

ABSTRACT

The purpose of this study is to numerically simulate water-vapor two-phase flow in ducts with variable cross-section. The homogeneous equilibrium model is used to describe the two-phase in a converging-diverging nozzle with the inlet vapor quality in the rage of $0.05< x <0.8$ and back pressure in the range of $P_b > 0.1$ atm. The flow passage is assumed to be adiabatic and frictional. Fluid properties are calculated basis on the thermodynamic tables. The governing equations are solved by flux-vector splitting method explicitly. The numerical results indicate that the vapor quality decreases along the nozzle before the location of the shock wave and it increases after that. In other words, the behavior of the nozzle depends on the inlet vapor quality. Therefore, the condensing behavior of the nozzle changes to the evaporating behavior when the flow passes over the shock wave with the inlet vapor quality greater than about 0.5. In addition, the pressure distribution along the nozzle with the inlet vapor quality of 0.73 is in a good agreement with the experimental data.

KEYWORDS:

Two-phase Flow, Homogeneous Equilibrium Model, Converging-Diverging Nozzle, Flux-Vector Splitting Method.

* Corresponding Author, Email: anouri@sharif.edu

1- INTRODUCTION

Vapor-liquid two-phase flow in ducts with variable cross-section has many industrial applications mainly in steam turbines, relief valves and expansion in the refrigeration systems. The models which are frequently used to simulate these flows are homogeneous equilibrium model, homogeneous non-equilibrium model and two fluid models. Homogeneous equilibrium model (HEM) neglects the slip condition between the phases by employing local thermodynamic equilibrium assumption. The system of equations for this model is closed by introducing an equation of state. In comparison with other models, the main advantage of HEM lies in the fact that no experimental constant or relation is needed to close the system of equations. However, the introduction of an equation of state that is valid in a wide range of states including liquid phase, vapor-liquid phase and vapor phase remains challenging.

Ihm and Kim [1] extended advection upstream splitting type (AUSMPW+) and RoeM schemes to compressible two phase flows at all speeds by adopting perfect gas law for gaseous phase and stiffened equation of state for liquid phase. Faccanoni et al. [2] solved HEM system of equations by using Roe scheme and assuming that both vapor and liquid phases are governed by stiffened gas equation of state. Hamidi et al. [3] numerically solved transonic two phase flow with shock in converging-diverging nozzle. They assumed that gas phase treats as an ideal gas and liquid phase is incompressible.

In this paper, flux vector splitting method is extended to solve the system of equations of two phase homogeneous equilibrium model in a variable cross section duct. Meanwhile, in order to employ a general equation of state which is valid in liquid phase and two phase liquid-vapor mixture and pure vapor phase, thermodynamic tables are used to determine thermodynamic properties.

2- GOVERNING EQUATIONS

By neglecting external heat transfer, the governing equations of two phase equilibrium model in a variable cross section duct are as follows:

$$\frac{\partial(SU)}{\partial t}+\frac{\partial F}{\partial z}=H \tag{1}$$

U, F and H are respectively vector of conservative variable, flux vector and source terms which are defined as:

$$U=\begin{bmatrix}\rho\\ \rho u\\ E\end{bmatrix} \tag{2a}$$

$$F=S\begin{bmatrix}\rho u\\ \rho u^2+p\\ u(E+p)\end{bmatrix} \tag{2b}$$

$$H=S\begin{bmatrix}0\\ p\dfrac{dLnS}{dz}-\dfrac{2f\rho|u|u}{D_H}\\ 0\end{bmatrix} \tag{2c}$$

Where p is pressure, ρ is density, $S=S(z)$ is the cross section area, u is velocity, D_H is the hydraulic diameter, $f=0.079/Re^{0.25}$ is the friction factor and E is the total energy per unit volume which is related to the specific internal energy as $E=\rho e+0.5\rho u^2$. A general equation of state in the form of $p=p[\rho(U),\ e(U)]$ is considered to close the system of equations.

Given the inlet vapor quality and pressure and back outlet pressure, boundary conditions for flow inside a converging-diverging nozzle are:

$$\begin{aligned}&p(0)=p_{in}\\ &x(0)=x_{in}\\ &p(L)=p_{back}\qquad if\ M_{out}<1\end{aligned} \tag{3}$$

3- NUMERICAL SOLUTION METHOD

Flux vector splitting method is extended to solve the system of Equations (1 - 2) numerically. Steger and Warming used this scheme to solve the gas dynamics equations for the first time [4].

Boundary conditions are implemented according to the sign of characteristic curves' slope which are equal to the eigenvalues of Jacobian matrix. At subsonic inlet pressure and vapor quality are specified and velocity is extrapolated from internal nodes. At subsonic outlet pressure is specified however velocity and density are extrapolated. All of the conservative variables are extrapolated at supersonic outlet.

4- RESULTS

In this research, two phase flow of water and vapor in a Deich nozzle [5] is investigated numerically. The geometry of the nozzle consists of a circular inlet section with a radius of 28 mm, followed by a cone with constant angle of aperture 3º and a length of 122 mm. The square root of the sum of the square of the pressure difference between two temporal consecutive values is taken as the convergence criteria as $\sqrt{\sum_{i=1}^{N}\left[\left|p_i^{n+1}-p_i^n\right|/p_i^n\right]^2}<10^{-8}$.

Figure 1 indicates the vapor quality distribution along the nozzle and associated shock wave locations. The inlet pressure and quality are 1.2 bar and 0.73 respectively for three outlet pressures of 0.55, 0.75 and 0.95 bar. We can see that the two-phase flow expansion in the divergent part of the nozzle is more significant than that of the convergent part. The flow conditions in Figure 1 is also reproduced in Figure 2 except for inlet quality of 0.17. The results illustrate that changing the inlet quality can change the behavior of the expansion process significantly.

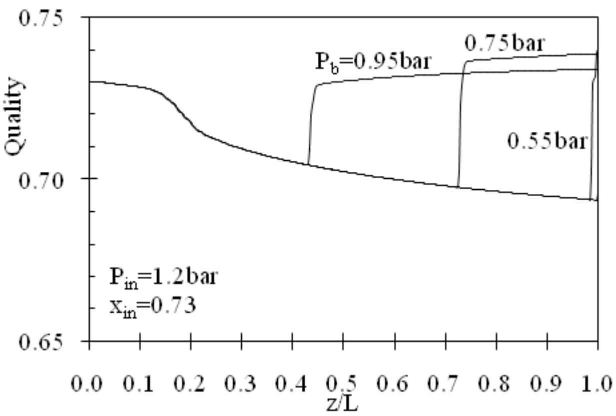

Figure 1. Vapor quality distribution along the nozzle associated with condensing behavior of the nozzle

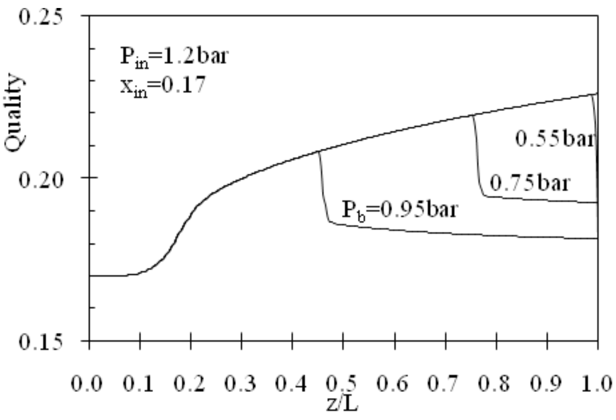

Figure 2. Vapor quality distribution along the nozzle associated with evaporating behavior of the nozzle

5- CONCLUSIONS

In this research, the numerical method of flux - vector splitting is extended to numerical simulation of homogeneous equilibrium two phase flows in a converging-diverging nozzle. The relations of the splitting of the flux vector and sound velocity are derived as a function of the thermodynamic properties of saturated liquid and vapor. Considering the numerical results, it can be concluded that flux vector splitting method has an acceptable accuracy in the prediction of shock wave location and vapor quality distribution along the nozzle. Numerical results indicate that the behavior of the vapor quality along the nozzle is substantially dependent on the inlet vapor quality in such a way that two phase flow with high inlet vapor quality (>0.5) shows condensing characteristic until the location of shock wave afterwards it begins to evaporate. However, at low inlet vapor qualities (<0.5) two phase flow evaporates until reaching the location of shock wave afterwards it condenses. In this case, vapor quality before the shock wave increases and consequently the two phase mixture becomes more compressible whereas its compressibility decreases after passing the shock wave. In addition, the pressure distribution along the nozzle with the inlet vapor quality of 0.73 is in good agreement with the experimental data. As the inlet vapor quality becomes relatively high, the difference between the numerical and experimental results decreases and the maximum error between them is about 5 percent and occurs immediately downstream of the throat.

6- REFERENCES

[1] Ihm, S.W., and Kim, C., "Computations of homogeneous-equilibrium two-phase flows with accurate and efficient shock-stable schemes", AIAA journal, Vol. 46, No. 12, pp. 3012– 3037, 2008.

[2] Faccanoni, G., Kokh, S., and Allaire, G., "Modelling and simulation of liquid-vapor phase transition in compressible flows based on thermodynamical equilibrium". ESAIM: Mathematical Modelling and Numerical Analysis, Vol. 46, No. 5, pp. 1029– 1054, 2012.

[3] Hamidi, S., Kermani, M.J., Beheshti, H., "Numerical Modeling of Transonic Two Phase Flow with Shock in Converging-Diverging Nozzle", Amirkabir Journal of Science & Research (Mechanical Engineering), Vol. 45, No. 1, pp. 15– 27, 2013.

[4] Pletcher, R. H., Tannehill, J. C., & Anderson, D., "Computational fluid mechanics and heat transfer", CRC Press, pp. 373– 378, 2012.

[5] Städtke, H., "Gasdynamic Aspects of Two-Phase Flow: Hyperbolicity, Wave Propagation Phenomena and Related Numerical Methods", John Wiley & Sons, pp. 188– 190, 2006.