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Numerical investigation of Joule heating effects on electroosmotic flow through a microchannel with triangular cross-section

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ABSTRACT

In this paper, numerical investigation of Joule heating effects on the electroosmotic flow through a microchannel with the triangular cross section and constant wall temperature have been presented. The energy equation for the temperature distribution, Navier–Stokes equation for the velocity distribution and a Poisson equation for the electric potential distribution have been solved by using the finite-volume method in a system curvilinear coordinates. Thermophysical properties such as the dynamic viscosity and electric conductivity vary with temperature. The results show that by increasing the Joule number, the temperature, velocity and mass flow rate increase with constant EDL number. With constant Joule number, the increments of EDL number causes the mass flow rate to increase. Mean temperature and velocity reduced by increasing the angle between sides and base of the cross-section in the particular Joule number.

KEYWORDS

Electroosmotic flow, Joule heating, Triangular microchannel, Constant wall temperature.

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1- INTRODUCTION

Microfluidics is one of the growing fields of micro electro mechanical systems due to its application in medical, biology sciences and analytical chemistry.

Generally it's shown that electroosmotic flows are generated by applying external electrical field on a charged layer, which is created by the interaction between ions in electrolyte solution and static charges on the walls. These two layers together are called twofold electrical layer (EDL).

On the other hand applying external electrical field on ions creates Joule heating. Thermal effects of this phenomenon can affect fluid's thermophysical properties; especially during biochemical activities this heat will be a significant source.

A survey of the relevant literature indicates that most investigations have been confined to electroosmotic flows but the effects of Joule heating on electro-osmotic flows in triangular micro channels with various section angle is not studied yet[1-5].

Analytical solution of electroosmotic flows in micro channels with triangular cross section is very difficult. In such flows, Naiver-Stokes equations for electrolyte solution flow are greatly coupled with energy equation and governed equation of electrical potential. In the present work, numerical techniques are used to study these effects by finite volume method in general coordinates of the section.

2- MATHEMATICAL MODEL

Figure 1 shows the geometry of a triangular micro channel with Cartesian coordinate located on one of its ends. Micro channel's height is assumed to be constant, so by determining Phi angle, the unique geometry will be specified. Micro channel's length is long enough that the flow considered hydrodynamically fully developed in it. In this study, flow is laminar and incompressible, so governed equations for the flow are

$$\mu\nabla^2 w + f_z = 0 \qquad (1)$$

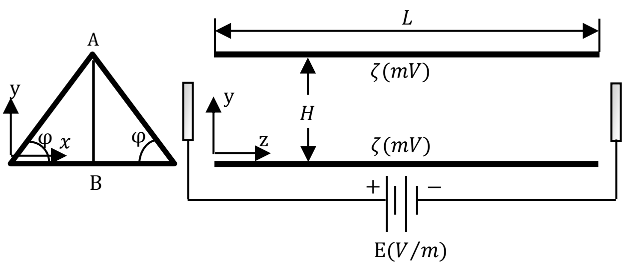

Fig. 1 Schematic of electroosmotic flow through triangular micro channel

Where f_Z is an electro-osmotic body force, which is caused by electro-kinetic effects on electrolyte solution, and equals to:

$$f_z = \rho_e E \qquad (2)$$

The relation between net electric charge density and electric potential in EDL can be expressed as below.

$$\nabla^2\psi = -\frac{\rho_e}{\varepsilon_0\varepsilon} \qquad (3)$$

For flows at the microscale, the local charge density can be approximated by the Boltzmann distribution.

$$\rho_e = 2n_\infty ze\sinh\left(\frac{-\psi}{k_bT/ze}\right) \qquad (۴)$$

It should be pointed out that when the electric potential is much smaller than the thermal energy of ion, i.e. Eq. (4) can be further simplified by the Debye–Hu¨ckle approximation, which gives

$$\rho_e = -\frac{2n_\infty z^2e^2\psi}{k_bT} \qquad (۵)$$

Equations 1 and 3 are temperature dependent, for solving them, first we should find temperature distribution of the solution. It is obvious that the solution temperature depends on Joule heat and heat transfer in electro-osmotic flow. Therefore, the energy equation is

$$\rho c_P w\frac{\partial T}{\partial z} = k(\nabla^2 T) + \dot{q} \qquad (6)$$

Where $\dot{q}$ is the electrical resistance heat source of electrolyte to the external electrical field, which is called Joule heating. Determining Joule heating by using Ohm's law as below

$$\dot{q} = \sigma E^2 \qquad (7)$$

Where sigma is the electrical conductivity of the fluid. Non-Dimensional form of the govern equations are

$$\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\kappa^2\Psi}{\eta(\theta+1)} = 0 \qquad (8)$$

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} - \frac{\kappa^2\Psi}{\theta+1} = 0 \qquad (9)$$

$$Pe\frac{\partial\theta}{\partial Z} = \frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2} + \beta^2\frac{\partial^2\theta}{\partial Z^2} + \frac{D_h^2E^2\sigma}{kT_0} \qquad (10)$$

In this study, dynamic viscosity and electrical conductivity are defined as temperature dependent as below, respectively.

$$\mu = 2.761 * 10^{-6}\exp\left(\frac{1713}{T}\right)$$

$$\sigma = \sigma_0(1 + 0.025(T - T_0))$$

For energy equation (10), we should mention that by considering assumptions $\frac{H}{L} \ll 1$ and $Pe \in (1-10)$, terms related to temperature changes in the flow direction are negligible despite the flow doesn't have thermal fully developed assumption. So, energy equation will be reduced to

$$\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2} + J_u(1 + \xi\theta) = 0 \qquad (11)$$

Calculations for obtaining non-dimensional temperature distribution start with solving equation 11 in order to determine Joule heating and it continues with equations 8 and 9 for determining velocity field and distribution of the electrical potential field, respectively. In each step of numerical solving of each equation, convergence condition for stopping the procedure is set as non-dimensional residual of each equation reaches 10^{-14}.

3- BOUNDARY CONDITIONS

According to ionic and hydrodynamically fully developed assumption in equations 8 and 9, there is no need to set boundary condition in inlet and outlet. No slip condition is considered for the walls and the temperature and electrical potential $\zeta\,(V)$ are constant on them. Therefore, the boundary conditions are

$$w = 0\,;\; T = T_0\,;\;\; \psi = \zeta \qquad (12)$$

4- RESULTS

Validating code is done by comparing the results from the numerical solution of electro-osmotic flow between two parallel plates with angle $\varphi \rightarrow 0$ and existed analytical solution of the same problem; and the results are shown with an accuracy order of 10^{-14}.

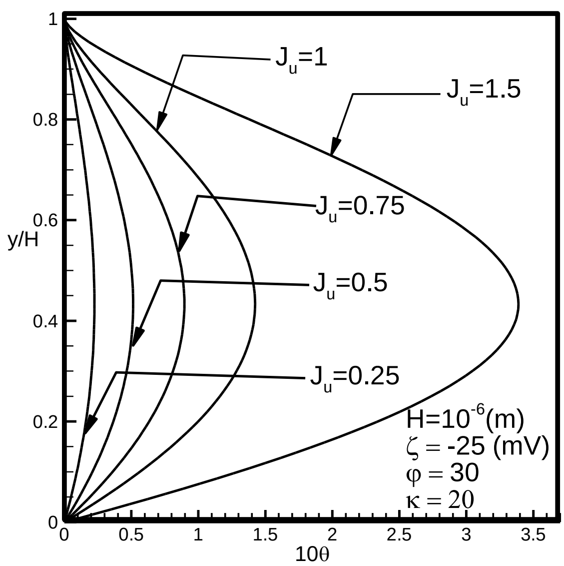

Fig. 2 Temperature distributions on line AB vs. joule number variations ($\kappa = 20, \varphi = 30$)

Figures 2 and 3 show temperature and velocity distributions on line AB of the micro channel section according to changes of Joule number in $\kappa = 20$ and $\varphi = 30$, respectively. As shown in figure 1, the growth of Joule number has great effect on increasing temperature in which non-dimensional maximum temperature for J_u is about 0.3 that means temperature at the center of the micro channel is 25% of walls temperature. It should be noticed that this temperature increase is defined on Joule number through increasing electrical field and with respect to energy equation; increasing E in source term caused temperature growth. As figure 2 shows, the velocity distribution has great growth with the increasing temperature.

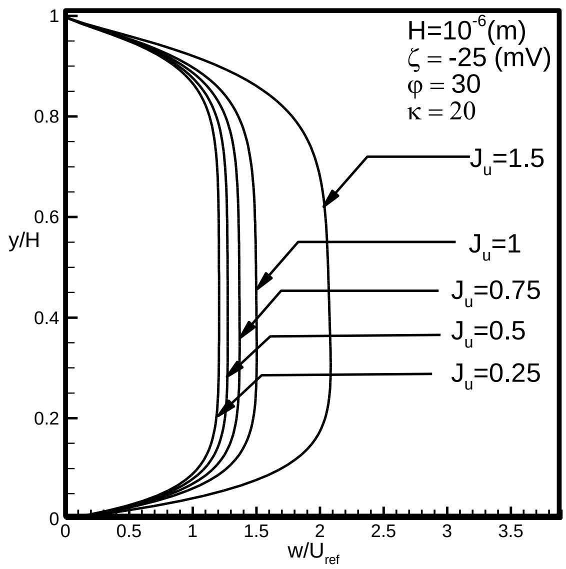

Fig 3. Velocity distributions on line AB vs. joule number variations ($\kappa = 20, \varphi = 30$)

5- CONCLUSION

The effects of Joule heating in electroosmotic flow through triangular micro channel are studied in this article. As it is shown in results considering Joule effects generates heat and this heat leads to the temperature growth. Increasing the temperature affects fluid's thermophysical properties while viscosity decreases exponentially and electrical conductivity increases linearly. Increasing Joule number causes the temperature growth, viscosity reduction and velocity and flowrate increase as a result in constant angle and EDL number while it has little effect on the distribution of electrical potential or in other words, body force.

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