AmirKabir University of Technology
(Tehran Polytechnic)

AmirKabir Jounrnal of Science & Research
Mechanical Engineering
(ASJR-ME)
Vol. 48, No. 2, Summer 2016, pp. 45-48

Vibration Analysis of an AFM Microcantilever with Sidewall and Top Surface Probes Based on the Couple Stress Theory

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(Received 14 March, 2013, Accepted 25 October, 2015)

ABSTRACT

In this paper, resonant frequency and sensitivity of vibration modes of an atomic force microscope with an assembled cantilever probe are analyzed utilizing the modified couple stress theory. The proposed ACP comprises a horizontal microcantilever, a vertical extension and two tips located at free ends of the cantilever and the extension, which make AFM capable of scanning top surface and the sidewall of the sample, simultaneously. To derive exact solution of the proposed ACP vibration behavior, the horizontal cantilever is modeled as two beams. Having obtained equation of motion, boundary and continuity conditions, the resonant frequency and sensitivity are studied. Results predicted by the current theory are compared to those obtained by the numerical methods presented by Kahrobaiyan etl al. [Kahrobaiyan, M.H., et al., 2010, "Sensitivity and resonant frequency of an AFM with sidewall and top-surface probes for both flexural and torsional modes", *International Journal of Mechanical Sciences*, 52 (10), pp. 1357-1365] and the comparison shows that there is some errors in results of numerical method. Results of the couple stress theory are also compared to those of the classical beam theory. Evaluations indicate that the vibration behavior of the proposed ACP is completely size-dependent.

KEYWORDS:

Atomic Force Microscope, Assembled Cantilever Probe, Modified Couple Stress theory, Size-dependent Behavior, Material Length Scale, Sensitivity.

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1- Brief Introduction

The atomic force microscope (AFM) is not only a powerful tool for imaging surface topography, but thanks to recent technical advances, it has proven to be the most frequently used scanning probe method for characterization, manipulation and modification of a variety of materials such as DNA, antibodies, polymers, and silicon surfaces .

To obtain information about the surface image and also morphologies and nanostructures of samples, dynamic behavior of the AFM cantilever must be fully investigated. The study of dynamic behavior of the AFM cantilever is currently attracting increasing interest [1].

In order to overcome the limitations of conventional AFMs in topography and imagination of sidewalls, Dai et al. [2-3] proposed assembled cantilever probes (ACPs) for direct and non-destructive sidewall measurement of nano- and microstructures. Several works have been dedicated to investigate the dynamic behavior of the AFM with ACPs [4].

In this work, size-dependent vibration behavior of an ACP proposed by Dia et al. [3] which comprises a horizontal cantilever, a vertical extension and two tips located at free ends of the cantilever and extension are analyzed based on the couple stress theory and utilizing exact solution method.

2- Analysis

An AFM probe is applicable for simultaneous topography at the top surface and sidewalls of microstructures with a horizontal cantilever, a vertical extension and two tips located at the free ends of the cantilever and the extension, named as assembled cantilever probe (ACP) shown in Figure 1.

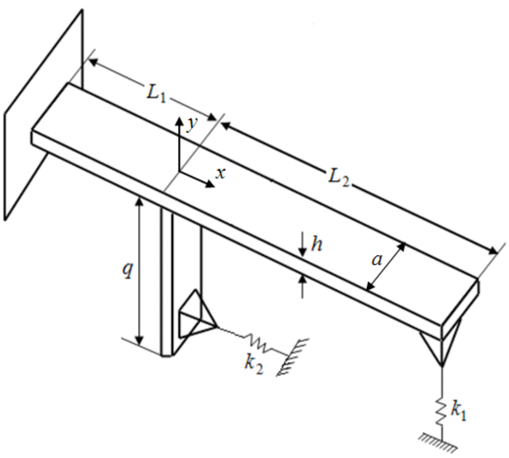

Figure 1. Schematic diagram of an AFM ACP cantilever

It is assumed that the extension is rigid. The ACP interaction with the sample surface at each tip are modeled by two springs k_1 and k_2. To investigate the vibration behavior of the proposed AFM ACP, the microcantilever is modeled as two beams. By modeling the cantilever as two beams, considering the potential and kinetic energy of the system according to the couple stress theory and utilizing the Hamilton principle, the governing equation and the continuity and boundary conditions of the proposed microcantilever are obtained. Afterwards, using complicated calculation, the characteristic equation of the system are found as follows.

$$D(\omega, \beta_1, \beta_2) = \det[A(\omega)] = 0 \tag{1}$$

While the non-zero components of A are given as follows:

$$
\begin{aligned}
&A_{11} = \sin \mu\tilde{L}_1, \quad A_{12} = -\sinh \mu\tilde{L}_1, \quad A_{13} = \cos \mu\tilde{L}_1, \quad A_{14} = \cosh \mu\tilde{L}_1, \\
&A_{21} = \cos \mu\tilde{L}_1, \quad A_{22} = \cosh \mu\tilde{L}_1, \quad A_{23} = \sin \mu\tilde{L}_1, \quad A_{24} = -\sinh \mu\tilde{L}_1, \\
&A_{35} = -(1+\lambda)\mu^2 \sin \mu\tilde{L}_2, \quad A_{36} = (1+\lambda)\mu^2 \sinh \mu\tilde{L}_2, \\
&A_{37} = -(1+\lambda)\mu^2 \cos \mu\tilde{L}_2, \quad A_{38} = (1+\lambda)\mu^2 \cosh \mu\tilde{L}_2, \\
&A_{45} = \beta_1 \sin \mu L_2 + (1+\lambda)\mu^3 \cosh \mu\tilde{L}_2, \\
&A_{46} = \beta_1 \sinh \mu L_2 - (1+\lambda)\mu^3 \cosh \mu\tilde{L}_2, \\
&A_{47} = \beta_1 \cos \mu L_2 - (1+\lambda)\mu^3 \sin \mu\tilde{L}_2, \\
&A_{48} = \beta_1 \cosh \mu L_2 - (1+\lambda)\mu^3 \sinh \mu\tilde{L}_2, \\
&A_{57} = A_{58} = A_{65} = A_{66} = -1, \quad A_{53} = A_{54} = A_{61} = A_{62} = 1, \\
&A_{71} = A_{72} = \mu\left(\frac{1}{3}\tilde{\rho}Q^3\omega^2 - \beta_2 Q^2\right), \\
&A_{73} = A_{78} = -A_{74} = -A_{77} = (1+\lambda)\mu^2, \\
&A_{81} = A_{86} = -A_{82} = -A_{85} = (1+\lambda)\mu^3, \\
&A_{83} = A_{84} = -\tilde{\rho}\omega^2
\end{aligned}
\tag{2}
$$

The dimensionless flexural sensitivity, S is defined as the differentiation of the dimensionless natural frequency with respect to the dimensionless surface contact stiffness.

To have a better comparison between the results obtained by the present theory and those predicted by the classic theory, a relative error percentage of the resonant frequency and sensitivity are considered.

3- Results

Figure 2 shows the sensitivity of the first three modes of sidewall tip predicted by analytical method and Rayleigh–Ritz method done by Kahrobaiyan et al. [4]. The relative error percentage of the first resonant frequency are depicted in Figure 3. Figure 4 shows the relative error percentage of the first sensitivities for the first tips.

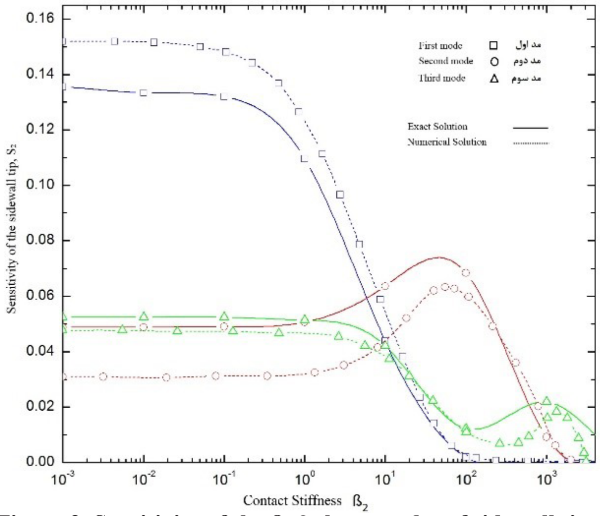

Figure 2. Sensitivity of the first three modes of sidewall tip

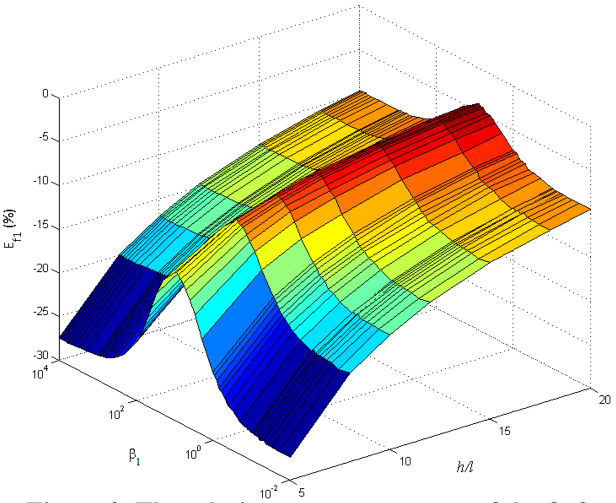

Figure 3. The relative error percentage of the first resonant frequency

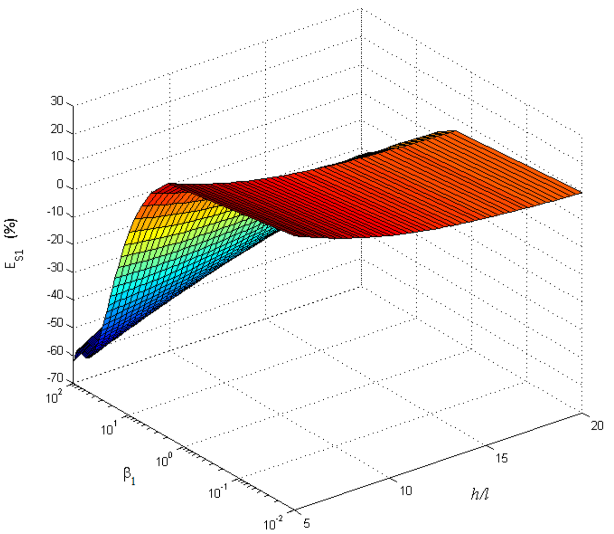

Figure 4. The relative error percentage of the first flexural sensitivity for the first tip

4- Conclusions

In this study, the governing equation, and boundary and continuity conditions of an AFM cantilever with sidewall probe are derived based on the modified strain gradient theory. Because the vertical sidewall is located between the clamped and free ends of the microcantilever, the cantilever is modeled as two beams. First, the resonant frequency and sensitivity of top surface tip and sidewall probe tip are analytically obtained and then compared to the results predicted by Rayleigh–Ritz method. It was found that the results predicted by Rayleigh-Ritz method are not reliable for high values of contact stiffness or when it is used to investigate the higher harmonic behavior. The results also declare that utilizing a non-classic theory is essential in analysis of the vibration behavior of the proposed ACP.

5- Main References

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