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## *Nonlinear Modeling and Sensitivity Analysis of a Double-Notch Hydraulic Engine Mount*

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### **ABSTRACT**

Hydraulic engine mounts are used to support powertrain and to reduce transmitted vibration from motors to the frame. One important use of hydraulic engine mounts is in aviation industry. In turbofans, maximum amplitude of vibration occurs at two distinct frequencies. So, by using an engine mount with maximum flexibility at these frequencies (called notch frequency), one can achieve reduction of transmission of engine induced vibrations to cabin. Here in this paper, using bond graph method, linear modeling of a double-notch hydraulic engine mount is done. Then, by applying nonlinear terms, nonlinear modeling of the engine mount is carried out using variance method. It has been shown that the dynamic stiffness will be changed by considering nonlinear terms. Finally, sensitivity analysis is applied to show the effect of each parameter on dynamic behavior of the mount. It has been shown that diameters of outer and inner inertia tracks are most effective parameters on the first and second notch frequencies, respectively.

### **KEYWORDS:**

Vibration Isolation, Hydraulic Engine Mount, Nonlinear Modeling, Dynamic Stiffness, Notch Frequency

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1- Introduction

Hydraulic engine mounts are used to connect motor and chassis structures together and to reduce vibration transmission from one to the other [1]. Hydraulic engine mounts are elastomeric mounts with fluid traveling between the top and the bottom rubber chambers (Figure 1).

A fluid channel between the two rubber chambers, called “inertia track”, is used to connect them together.

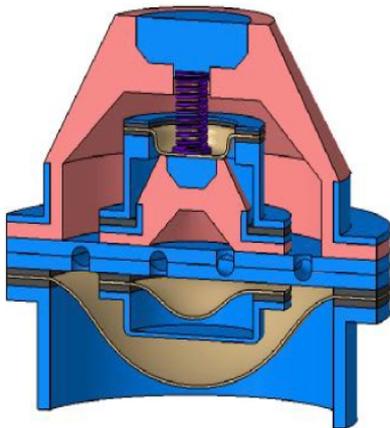


Figure 1. Double-notch hydraulic engine mount

Resonance caused by the travelling fluid in the inertia track can create a tuned vibration absorber effect to provide vibration isolation at a particular frequency called notch frequency [2]. One important use of hydraulic engine mounts is in the aviation industry. In turbofans, the maximum amplitude of vibration occurs at the two distinct frequencies. So, by using an engine mount with maximum flexibility at these frequencies, transmission of the engine induced vibrations to the cabin can be reduced.

Here, a double-notch hydraulic engine mount design is proposed that uses two working fluids. This new design has two notch frequencies and two peak frequencies, when viscosities of both working fluids are kept low.

Linear modeling of this mount is done by Tikani et al. [3]. As nonlinear terms are neglected in previous studies, here, the modeling is carried out by considering these terms.

Here, in this paper, linear modeling of a double-notch hydraulic engine mount is done using bond graph method. Then, by considering nonlinear terms, nonlinear modeling of the engine mount is carried out using variance method. Finally, sensitivity analysis is applied to show the effect of each parameter on the dynamic behavior of the mount.

2- Double-Notch Hydraulic Engine Mount Model

The bond graph model of double-notch hydraulic engine mount is shown in Figure 2. The state space equations, from the bond graph model, can be obtained as follows:

$$\dot{q}_2 = V_{in} \tag{1}$$

$$\dot{q}_6 = A_{po}V_{in} - \frac{P_8}{I_8} - A_i \frac{P_{16}}{I_{16}} \tag{2}$$

$$\dot{p}_8 = \frac{q_6}{C_6} - R_9 \frac{P_8}{I_8} - \frac{q_{10}}{C_{10}} \tag{3}$$

$$\dot{q}_{10} = \frac{P_8}{I_8} \tag{4}$$

$$\dot{q}_{12} = V_{in} - \frac{P_{16}}{I_{16}} \tag{5}$$

$$\dot{p}_{16} = A_i \frac{q_6}{C_6} + \frac{q_{12}}{C_{12}} - R_{17} \frac{P_{16}}{I_{16}} - \frac{q_{18}}{C_{18}} - A_{pi} \frac{q_{21}}{C_{21}} \tag{6}$$

$$\dot{q}_{18} = \frac{P_{16}}{I_{16}} \tag{7}$$

$$\dot{q}_{21} = A_{pi} \frac{P_{16}}{I_{16}} - \frac{P_{23}}{I_{23}} \tag{8}$$

$$\dot{p}_{23} = \frac{q_{21}}{C_{21}} - R_{25} \frac{P_{23}}{I_{23}} - \frac{q_{24}}{C_{24}} \tag{9}$$

$$\dot{q}_{24} = \frac{P_{23}}{I_{23}} \tag{10}$$

$$F_T = \frac{q_2}{C_2} + R_3V_{in} + A_{po} \frac{q_6}{C_6} + \frac{q_{12}}{C_{12}} \tag{11}$$

To simulate the following state space equations MATLAB program with the parameters reported in Tikani et al. study [3] were used.

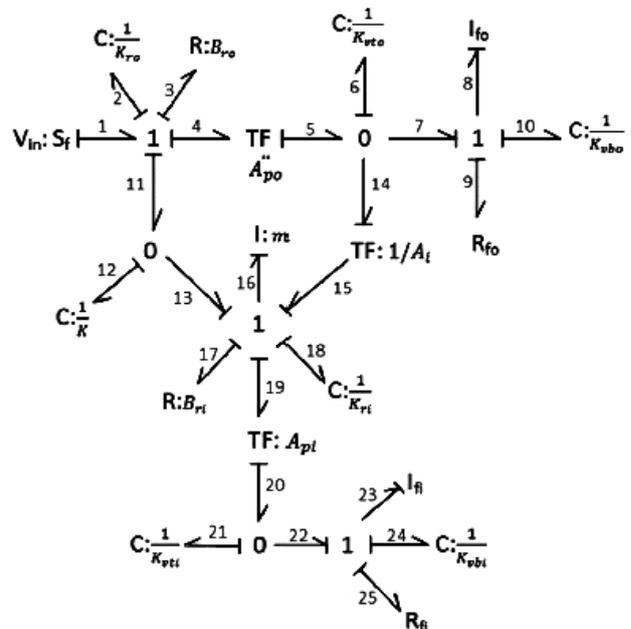


Figure 2. Bond graph model of a double-notch hydraulic engine mount [3]

3- Nonlinear Modeling

In order to consider nonlinear terms, head loss coefficients in the inertia tracks are imported in the

model. Nonlinear fluid friction in the inertia track can be obtained as:

$$R'_f = \frac{\rho}{2A^2} \sum K \tag{12}$$

where  $K$  is the sum of head loss coefficients.

Based on this term, Eqs. (3) and (9) can be rewritten as follows:

$$\dot{P}_8 = \frac{q_6}{C_6} - R_9 \frac{P_8}{I_8} - \frac{q_{10}}{C_{10}} - R_{26} \frac{P_8}{I_8} \left| \frac{P_8}{I_8} \right| \tag{13}$$

$$\dot{P}_{23} = \frac{q_{21}}{C_{21}} - R_{25} \frac{P_{23}}{I_{23}} - \frac{q_{24}}{C_{24}} - R_{27} \frac{P_{23}}{I_{23}} \left| \frac{P_{23}}{I_{23}} \right| \tag{14}$$

Using variance method, the frequency response of the system is calculated. In this method, the dynamic stiffness at a fixed frequency can be obtained as:

$$K^*(\omega) = \sqrt{\frac{\int_0^T (F(t))^2 dt}{\int_0^T (X(t))^2 dt}} \tag{15}$$

Figure 3 compares the simulations of linear and nonlinear modeling.

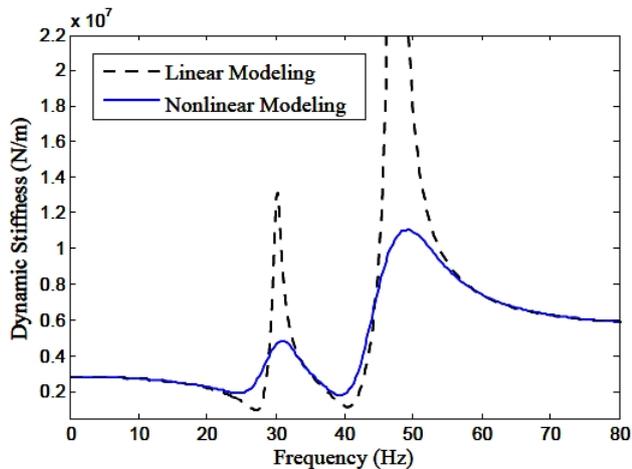


Figure 3. Dynamic stiffness of hydraulic engine mount

#### 4- Sensitivity Analysis

In this section, the variation of each hydraulic engine mount parameter on the location of the notch frequencies and dynamic stiffness at these frequencies is studied. In this analysis, each parameter of the

Table 1. Sensitivity analysis results by 20% change in the mount parameters

Parameters	$f_{N1}$	$f_{N2}$	$K^*_{N1}$	$K^*_{N2}$
$L_o$	-4.3 %	-2.6 %	-5.1 %	+6.7 %
$L_i$	-4.3 %	-7.7 %	+0.7 %	-11.1 %
$D_o$	+17.4 %	+5.1 %	-1.6 %	-14.6 %
$D_i$	0 %	+15.4 %	-1.2 %	+16.1 %
$K_{ro}$	0 %	0 %	+22.4 %	+14.9 %
$K_{ri}$	+4.3 %	0 %	-1.5 %	+9.5 %
$K_{vo}$	0 %	0 %	-1.2 %	-7.3 %
$K_{vi}$	0 %	+2.6 %	0 %	+2 %
$\rho_o$	-8.7 %	-2.6 %	-1.8 %	+8 %
$\rho_i$	-4.3 %	-7.7 %	+0.7 %	-9.2 %

mount is varied by 20% and the percent change on the location of the notch and dynamic stiffness is recorded. Table 1 illustrates the results.

Table 1 indicates that the diameters of outer and inner inertia tracks are most effective parameters on the first and second notch frequencies, respectively.

#### 5- Conclusion

In this paper nonlinear modeling of a double-notch hydraulic engine mount is studied. It has been shown that the dynamic stiffness will be changed by considering nonlinear terms. By applying sensitivity analysis, the effect of each parameter is calculated and most-effective parameters are determined.

#### 6- References

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