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Thermal Non-Equilibrium Similarity Solution for Nanofluid Boundary Layer in a Porous Medium

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ABSTRACT

In the present study, we have investigated the external free convection of a nanofluid near a vertical heated surface embedded in a saturated porous medium using the thermal non-equilibrium assumption. The vertical surface has a linear temperature distribution with a uniform mass suction or injection. Assuming the Brownian motion and thermophoresis as the primary driving mechanisms of free convection of the nanofluid, suitable volume averaged equations are employed. We have also followed similarity solution method for transforming the governing equations in to the ordinary differential equations. The new set of ordinary equations is solved numerically by the Shooting method and the flow and temperature fields are determined completely. The obtained numerical results are employed for calculating the Nusselt numbers for both the solid and liquid phases in the physical domain. Moreover, the Sherwood number for the nanoparticles is determined over a wide range of parameters.

KEYWORDS:

Similarity Solution, Natural Convection, Nanofluids, Thermal non-Equilibrium.

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1- Introduction

Convective heat transfer in nanofluids is a major interest topic in the heat transfer research community. Also the study of the natural convection flow and heat transfer from surfaces which are held at a temperature that is different from the ambient porous medium has been of considerable interest in energy-related engineering problems for many decades.

In the present paper we consider the combined effects of Local Thermal Non-Equilibrium (LTNE), uniform surface suction/blowing and buoyancy due to the presence of linear variations of the temperature. The similarity solutions have to be solved numerically, and this forms the focus of the present paper.

Our work extends previous papers by Khan and Aziz [1] and Kuznetsov and Nield [2]. Local thermal non-equilibrium is modeled by two separate equations of heat transport, one for the fluid phase and one for the solid phase.

2- Governing Equations

We consider a vertical plate at *y=0*. At this plate the temperature *T* and the nanoparticle fraction *φ* take values $T_w=T_\infty+Ax$ and $\varphi_w=\varphi_\infty+Bx$, respectively. Here, *A* and *B* are positive constants.

The plate may be permeable or impermeable. The fluid properties are assumed to be constant except for density variations in the buoyancy force term. The governing equations allow for the presence of local thermal non-equilibrium effects.

The boundary layer equations for the flow of an incompressible viscous nanofluid are given as,

$$\begin{aligned}
& f'' = \theta_f' - NZ' \\
& \theta_f'' + H\left(\theta_s - \theta_f\right) = f'\theta_f - f\theta_f' - NbZ'\theta_f' \\
& \qquad\qquad\qquad\qquad - Nt\theta_f'^2 \\
& \theta_s'' + H\gamma\left(\theta_f - \theta_s\right) = 0 \\
& Z'' = -Le(f\,Z' - f'\,Z) - \frac{Nt}{Nb}\theta_f'' \qquad (1)
\end{aligned}$$

The thermal conductivity ratio and the non-dimensional heat transfer coefficient are defined respectively as $\gamma=\varepsilon k_f/(1-\varepsilon)k_s$ and $H=hx^2/\varepsilon k_f\,Ra_{XT}$ where $Ra_{XT}=g\,K\,\beta\,A\,x2/(\nu\,\varepsilon\,\alpha_f)$.

The parameters *N* (nanofluid buoyancy-ratio), *Nb* (Brownian motion parameter) and *Nt* (thermophoresis parameter) are,

$$\begin{aligned}
& Nt = \frac{(\rho c_p)_{particle} D_T (T_w - T_\infty)}{\alpha_f (\rho c_p)_f T_\infty}, \\
& Nb = \frac{(\rho c_p)_{particle} D_B (\phi_w - \phi_\infty)}{\alpha_f (\rho c_p)_f}, \qquad (2) \\
& N = \frac{\beta_\phi (\phi_w - \phi_\infty)}{\beta_T (T_w - T_\infty)}
\end{aligned}$$

The boundary conditions are defined as,

$$\begin{aligned}
& \theta_f(\eta=0)=1\,,\ \theta_f(\eta=\infty)=0 \qquad (3) \\
& \theta_s(\eta=0)=1\,,\ \theta_s(\eta=\infty)=0 \\
& Z(\eta=0)=1\ ,\ Z(\eta=\infty)=0 \\
& f(\eta=0)=f_w,\ f'(\eta=\infty)=0
\end{aligned}$$

3- Results

The variations in the local Nusselt numbers with *H* are shown in Figure 1 for different values of f_w. The three different frames correspond to three different values of the suction/injection parameter.

The approach to LTE is seen clearly as *H* increases as the Nusselt number curves for the two phases approach one another. At the opposite extreme, as *H* decreases, the Nusselt numbers for the fluid phase tend towards a common value which is independent of f_w. Moreover, a decrease in *H* also means that the boundary layer thickness of the solid phase temperature increases, which corresponds to the Nusselt number tending towards zero in that limit.

In addition, the variations in the local Nusselt numbers with *H* are shown in Figure 2 for different values of *Nb* and *Nt*.

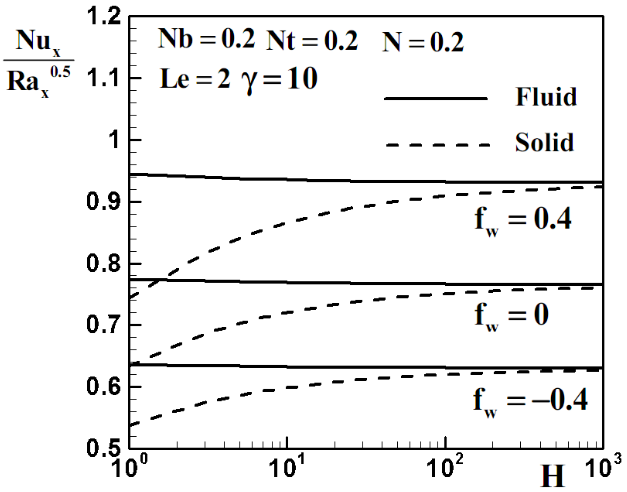

Figure 1. Variation of the Nusselt numbers for different values of f_w

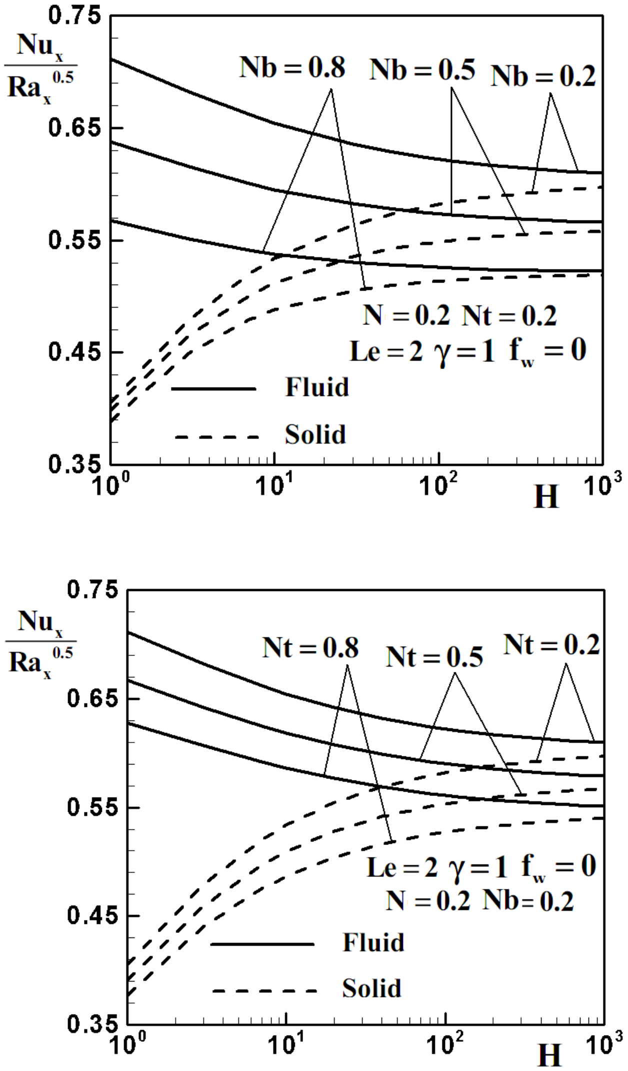

Figure 2. Variation of the Nusselt numbers with *H* for different values of *Nb* and *Nt*.

4- References

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