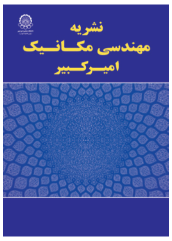

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Investigation of Surface Effects on Free Torsional Vibration of Nanobeams

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ABSTRACT: In this study, surface effects on the free torsional vibration of nanobeams are investigated. To this end, equations of motion of nanobeams incorporating the surface effects are derived using the Hamilton's principle and based on the surface elasticity theory. Then, equations of motion are analytically solved for three types of boundary conditions: clamped-clamped, clamped-free, and free-free, and associated mode shapes and natural frequencies are obtained. Nanobeams made of aluminum and silicon are selected as case studies. A detailed study is performed to examine the surface effects on the free torsional vibration of nanobeams for various nanobeam lengths, nanobeam radii, and mode numbers. In addition, influences of each of the surface parameters on torsional natural frequencies are separately investigated. The results show that influences of the surface effects on the free torsional vibration of nanobeams are completely different from those on the free transverse vibration of nanobeams. Results of the present study can be useful in design of nano-electro-mechanical systems like nano-bearings and rotary servomotors.

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1- Introduction

The small scale and surface effects on mechanical behavior of nano-structures are considered in many researches with the aid of nonlocal theory of Eringen [1] and the surface elasticity theory [2], respectively. Torsional vibration of nano-structures is an important mechanical behavior that should be considered in design of nano-electro-mechanical systems like nano-bearings and rotary servomotors. For this reason, the aim of the present study is to investigate the surface effects on free torsional vibration of nanobeams. To this end, governing equations of motions of nanobeams are derived based on the surface elasticity theory by considering all surface parameters including surface density, surface stress, and surface shear modulus. Then, governing equations are solved analytically and natural frequencies are extracted for various boundary conditions.

2- Problem Formulation

Consider a nanobeam with length L ($0 \le x \le L$) and radius R (see Figure 1).

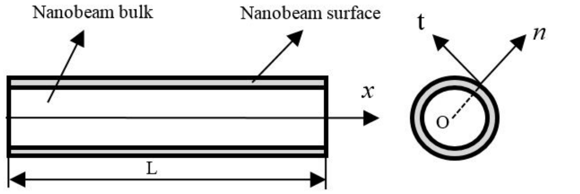

Figure 1. Schematic of nanobeam with surface effects

Due to rotation of cross section about center O, the displacement field at any point of the nanobeam can be written as [3]:

$$u_x(x,T)=0; \quad u_t(x,T)=r\theta(x,T); \quad u_n(x,T)=0 \tag{1}$$

where $\theta(x,T)$ is the rotational displacement of the nanobeam about center O. The stresses in the nanobeam are assumed to be
where G is the shear modulus of the nanobeam.

$$\sigma_{xt}=G\varepsilon_{xt}=Gr\frac{\partial\theta}{\partial x} \tag{2}$$

$$\sigma_{xx}=\sigma_{tt}=\sigma_{nn}=\sigma_{xn}=\sigma_{tn}=0 \tag{3}$$

Equations (2) and (3) are the nanobeam bulk stresses. In order to calculate the nanobeam surface stresses, the surface elasticity theory is implemented as follows [2]:
where $\tau_{\alpha\beta}^{\pm}$ are the surface stresses, $u_{\alpha,\beta}^{\pm}$ are the

$$\tau_{\alpha\beta}^{\pm}=\tau_0^{\pm}\delta_{\alpha\beta}+(\mu_0^{\pm}-\tau_0^{\pm})(u_{\alpha,\beta}^{\pm}+u_{\beta,\alpha}^{\pm}) + (\lambda_0^{\pm}+\tau_0^{\pm})u_{\gamma,\gamma}^{\pm}+\tau_0^{\pm}u_{\alpha,\beta}^{\pm}$$
$$\tau_{\alpha n}^{\pm}=\tau_0^{\pm}u_{n,\alpha}^{\pm} \tag{4}$$

surface strains, $\tau_0^{\pm}$ are residual surface tensions under unconstrained conditions, $\mu_0^{\pm}$ and $\lambda_0^{\pm}$ are the surface Lame´ constants, $\delta_{\alpha\beta}$ the Kronecker delta, $u_{\alpha}^{\pm}$ are displacement components of the surfaces $S^{\pm}$, and α and $\beta = x, t$. Assuming the same material properties for the layers of the nanobeam and considering the displacement components of the nanobeam, Equation 1, results in the surface stresses. Now, by using Hamilton's principle, the governing equations of motion and the boundary conditions of the nanobeam can be derived as:

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$$\alpha^2 \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 \theta}{\partial T^2} \quad (5)$$

$$\left(GI_p\right)^* \frac{\partial \theta}{\partial x} \delta\theta \Big|_0^L = 0 \quad (6)$$

where

$$\alpha^2 = \left(GI_p\right)^* / \left(I_0\right)^*;$$

$$\left(GI_p\right)^* = 2\pi R^3 \left(\mu_0 - \tau_0\right); \left(I_0\right)^* = 2\pi R^3 \rho_0 \quad (7)$$

$$I_0 = \rho I_P = \rho \iint_A \left(r^2\right) dA = \rho \pi R^4 / 2 \quad (8)$$

and I_0 is the mass moment of inertia of the nanobeam per unit length. Now, by considering harmonic rotational displacement as $\theta(x,t)=\Theta(x)^{i\omega T}$ and using Equations (5) and (6), the rotational mode shapes and natural frequencies of nanobeam for three different boundary conditions can be obtained as

- mode shapes

$$\text{CC:} \quad \Theta_n(x) = A_n Sin\left(n\pi x / L\right) \quad (9)$$

$$\text{CF:} \quad \Theta_n(x) = A_n Sin\left((2n-1)\pi x / L\right) \quad (10)$$

$$\text{FF:} \quad \Theta_n(x) = A_n Cos\left(n\pi x / L\right) \quad (11)$$

- natural frequencies

$$\text{CC:} \quad \omega_n = \frac{n\pi\alpha}{L} = \left(\frac{n\pi}{L}\right)\sqrt{\left(GI_p\right)^* / \left(I_0\right)^*} \quad (12)$$

$$\text{CF:} \quad \omega_n = \left(\frac{(2n-1)\pi}{2L}\right)\sqrt{\left(GI_p\right)^* / \left(I_0\right)^*} \quad (13)$$

$$\text{FF:} \quad \omega_n = \frac{n\pi\alpha}{L} = \left(\frac{n\pi}{L}\right)\sqrt{\left(GI_p\right)^* / \left(I_0\right)^*} \quad (14)$$

3- Results and Discussion

In this section, the surface effects on mode shapes and natural frequencies of nanobeams under torsion are investigated. As seen from Equations (9-11), the surface effects do not have any effects on torsional mode shapes of nanobeams for all types of boundary conditions. To consider the surface effects on torsional frequencies of nanobeams frequency ratio (*FR*) is defined as:

$$\text{Frequency ratio (FR)} = \frac{\text{Natural frequency with the surface effects}}{\text{Natural frequency without the surface effects}} \quad (15)$$

The first point obtained from Eqs. (12)-(14) is that the surface effects have the same influences on both CC and FF torsional natural frequencies.

In Table 1 the surface effects on torsional natural frequencies of nanobeams are listed for various nanobeam lengths and mode numbers. It is concluded that: 1) the surface effects have a decreasing effect on torsional natural frequencies for all types of boundary conditions and nanobeam materials; 2) the surface effects on torsional natural frequencies are independent of the mode number and nanobeam length; 3) the surface effects have the same effects on torsional natural frequencies of both CC and CF boundary conditions. Since it has been concluded from Eqs. (12)-(14) that the surface effects have the same influences on both CC and FF torsional natural frequencies, it can be said that the surface effects are independent of the nanobeam boundary condition type. It is worth to note that among boundary conditions CC, CF, and SS the surface effects have increasing influence on transverse natural frequencies of nanobeams with only CF end condition [4].

Table 1. Frequency ratios of al and si nanobeams for various mode numbers, lengths, and boundary conditions

Length (nm)	Mode number	CC		CF	
		Al	Si	Al	Si
5	1	0.205	0.658	0.205	0.687
	2	0.205	0.658	0.205	0.658
	3	0.205	0.658	0.205	0.658
10	1	0.205	0.658	0.205	0.658
	2	0.205	0.658	0.205	0.658
	3	0.205	0.658	0.205	0.658

Results reconfirm that that the surface effects have a decreasing effect on torsional natural frequencies, and its decreasing effect on natural frequencies of Al nanobeam is higher than those of Si nanobeam but also shows that the surface effects on *FRs* are dependent on the nanobeam radius. Results also show that as the nanobeam radius increases, the surface effects decrease. It can be observed that among the surface parameters the surface stress has the lowest decreasing effect on the torsional frequencies in comparison with the other surface parameters. It is also seen that both the surface density and the surface Lamé constant μ_0 have decreasing effects on torsional natural frequencies with different values. Furthurmore, it is seen that for Si nanobeam the decreasing effect of the surface Lamé constant μ_0 on natural frequencies is lower than the decreasing effect of the surface density while it is the other way round for Al nanobeam. A final point to note is that effects of the surface parameters do not obey the superposition principle.

4- Conclusions

Free torsional vibration of nanobeams with surface effects is studied in the present work. Governing equations of motions are derived based on the surface elasticity theory and are solved analytically. Then natural frequencies and mode shapes are obtained. The results show that the surface stress has the lowest effect on torsional natural frequencies of nanobeams among the surface parameters. It is also observed that the surface density and the surface Lamé constant μ_0 have decreasing effects and their effects are independent of the mode number and nanobeam length. It is also observed that the surface effects are dependent on the nanobeam radius and its effects decrease by increasing the nanobeam radius. Another interesting point is that the surface effects are independent of the nanobeam boundary conditions.

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