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Static Response of Smart Beams Equipped with Extension/Shearing Piezoelectric Patches Considering Poisson's Effect Based on Different Theories

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ABSTRACT

This paper deals with effect of Poisson's ratio on static response of smart laminated composite beams. Actuation performance of smart beams with extension and shear mode piezoelectric orientation has been investigated. The first order and high order shear deformation beam theory have been implemented for modelling the problem. Piezoelectric stress resultants are expressed in terms of Heaviside discontinuity functions. The state space approach is used to obtain an analytical solution for the response of smart beams with different boundary conditions. The effects of actuator length, location of piezoelectric patches, order of theory, fibre orientation, boundary condition and Poisson's ratio on the beam deflection, have been investigated. Also the response of beam with complete piezoelectric layer has been obtained with the same procedure and compared with the solution for the beams with piezoelectric patch of same length. Results show that neglecting Poisson's effect in calculations causes significant errors in the response of angle ply beam with extension mode actuators.

KEYWORDS:

Analytical Solution; Different Boundary Condition; Extension/Shear Mode; Piezoelectric Patch; Poisson's Effect.

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1- Introduction

Sensing and actuation applications have been expanded by development of piezoelectric materials in recent years. Great properties of these materials in comparison with usual materials make them very useful for application in sensors and actuators. Piezoelectric patches are being increasingly utilized in many structures such as aerospace applications, sporting goods, and MEMS [1].

In this paper, the effect of Poisson ratio on static response of smart laminated composite beam is investigated and also actuation performance of smart beams with extension and shear mode piezoelectric orientation has been studied. First a formulation based on the first-order and high order shear deformation beam theory for laminated composite is presented. The governing equations are derived using stationary potential energy with regard to the first and higher order displacement field; afterwards these equations are solved with the space state approach. Subsequently, a numerical example for the problem is investigated and the results are shown in the tabular and graphical form comparing the effect of different boundary conditions, actuator length, location of piezoelectric patches, fiber orientation, and Poisson ratio on beams deflection. Further, the results are compared to the deflection of a composite beam with full piezoelectric layer and conclusions of this paper have been fully discussed.

2- Main Formulation

According to higher order shear deformation theory, displacement field is introduced as follows [2]:

$$u(x,z) = u_0(x) + z\varphi(x) + a_2\left(\frac{z}{h}\right)^3 [w'(x) + \varphi(x)] \qquad w(x,z) = w_0(x) \tag{1}$$

where U and W are displacements along x and z axis. φ indicates the rotation of a plane normal to x-axis about the y-axis and w is the transverse displacement of the beam mid-plane, governing equations using principle of stationary potential energy are expressed as follows; a prime index on a variable means differentiation with respect to x [1]:

$$\varphi'' = c_1(\varphi + w') + c_2 w''' + f_1\left((M_x^p)' - Q_x^p\right) + f_2(P_x^p)' + f_3 S_x^p$$
$$w^{IV} = c_3(\varphi' + w'') + c_4\varphi''' + f_4(P_x^p)'' + f_5(Q_x^p)' + f_6(S_x^p)' \tag{2}$$

Stress resultants are expressed as:

$$\left(Q_x^p, S_x^p\right) = \int_{-h/2}^{h/2} \bar{Q}_{55} d_{15} E_1(1, z^2) dz \times [H(x - L_1) - H(x - L_2)]$$
$$\left(M_x^p, P_x^p\right) = \int_{-h/2}^{h/2} \bar{Q}_{11} d_{31} E_3(z, z^3) dz \times [H(x - L_1) - H(x - L_2)] \tag{3}$$

Q is the components of reduced stiffness matrix and H is Heaviside function. The governing equations are solved using state space approach.

In the case of considering the poison’s effect, the normal and shear stress in y direction in each layer must be zero [3]:

$$\sigma_y = Q_{12}\varepsilon_x + Q_{22}\varepsilon_y + Q_{26}\gamma_{xy} = 0$$
$$\tau_{xy} = Q_{16}\varepsilon_x + Q_{26}\varepsilon_y + Q_{66}\gamma_{xy} = 0$$
$$\tau_{yz} = Q_{44}\gamma_{yz} + Q_{45}\gamma_{xz} = 0 \tag{4}$$

Substation ε_y , γ_{xy} and γ_{yz} in the stress-strain relations in the x directions with Poisson’s effect, the following relation obtained:

$$\bar{Q}_{11} = Q_{11} + Q_{12}\frac{Q_{16}Q_{26} - Q_{12}Q_{66}}{Q_{22}Q_{66} - Q_{26}^2} + Q_{16}\frac{Q_{12}Q_{26} - Q_{22}Q_{16}}{Q_{22}Q_{66} - Q_{26}^2}$$
$$\bar{Q}_{55} = Q_{55} - \frac{Q_{45}}{Q_{44}} \tag{5}$$

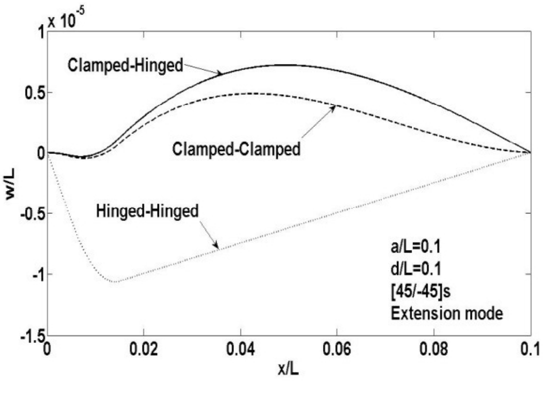
3- Main Results

Response of laminated composite beam has been investigated using different parameters and compared together.

The effects of different boundary conditions on the deflected shape of the beam are illustrated in Figure 1 for extension and shearing modes.

Figure 2 demonstrates the difference of first and high order theories results for the extension and shearing modes.

Table 1 shows the effect of Poisson ratio on the maximum deflection of angle ply beam, with extension mode actuator.



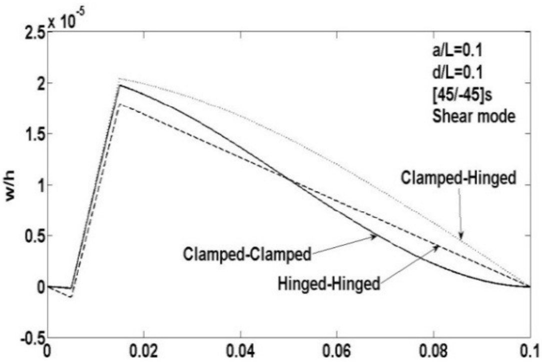


Figure 1. different boundary conditions effect

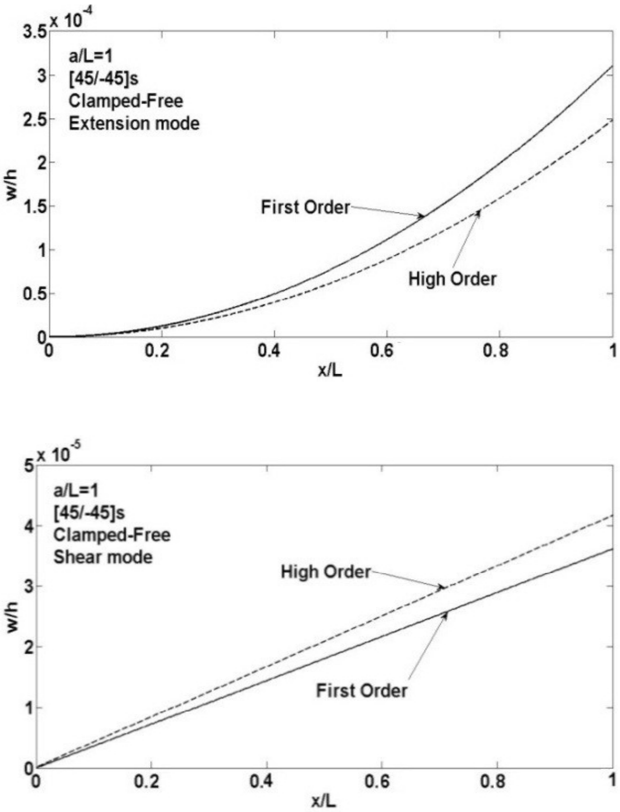


Figure 2. difference of first and high order theories results in extension and shearing modes.

Table 1. Poisson's effect on maximum deflection of an angle ply beam with extension mode actuator patch with clamped- free boundary condition

Layout	M0E0	M0E1	M1E0	M1E1
$[0/90]_s$	1.11e-4	1.04e-4	1.11e-4	1.01e-4
$[15/-15]_s$	1.11e-4	1.04e-4	1.21e-4	1.14e-4
$[30/-30]_s$	1.45e-4	1.38e-4	2.25e-4	2.17e-4
$[45/-45]_s$	2.13e-4	2.04e-4	3.15e-4	3.10e-4
$[60/-60]_s$	2.95e-4	2.89e-4	3.39e-4	3.35e-4
$[75/-75]_s$	3.38e-4	3.42e-4	3.34e-4	3.39e-4

4- Conclusion

An analytical solution for the static analysis of smart beams with extension and shear mode actuators is developed for various boundary conditions. The state-space concept in conjunction with the Jordan canonical form is used to obtain the solution by making use of Heaviside discontinuity functions. The actuation behavior of the beam with extension/ shear mode actuators is compared with different boundary conditions. The effects of Poisson ratio, boundary condition, fiber orientation, actuator length and location of it on the deflected shapes have been studied. Results show that Poisson ratio is negligible in cross ply beams, but it has major effects in angle ply beams. Also the effects of actuator length and its location on the response of beam have beam showed.

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