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Nonlinear Buckling Analysis of Nonlocal Boron Nitride Timoshenko Nano Beam Based on Modified Couple Stress Theory Using DQM

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ABSTRACT: In this article, nonlinear buckling analysis of nonlocal boron nitride Timoshenko nano beam on elastic foundation based on modified couple stress theory, nonlocal elasticity Eringen's model, and Von Karman nonlinear geometry theory are investigated. The governing equation of motion and boundary conditions based on Hamilton's principle are obtained. To solve the nonlinear governing equation of motion, the differential quadrature method is used to obtain the critical buckling load for two edges simply supported (S-S) and simply supported-clamped (S-C) boundary conditions. The results of this research are compared with the obtained results by other researchers and there is a good agreement. Finally, effects of various parameters such as nonlocal Eringen's parameter, slenderness ratio of nano beam, electric field, temperature changes and material length scale parameter on the nonlocal critical buckling load of Timoshenko nano beam are examined. The results show that with increasing nonlocal parameter, slenderness ratio, electric field, and temperature changes, the critical buckling load decreases. Meanwhile, the critical buckling load for S-S boundary condition is lower than that of for S-C.

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1- Introduction

Eringen's nonlocal elasticity theory [1] assumes that the stress at a point x in a body depends not only on the strain at that point but also on those at all points of the body. Thus, the equations change and the classical equations can't be used in this case. This theory has been widely used to analyze bending, buckling and free vibration of beams in micro- and nano-scale under electro-thermo-mechanical loading. Reddy [2] presented bending, buckling and vibration analysis of beams using nonlocal elasticity theory. Buckling analysis of a SWCNT embedded in an elastic medium was studied by Murmu and Pradhan [3] based on nonlocal elasticity and Timoshenko beam theory. In other work, Yang et al. [4] analyzed nonlinear free vibration of SWCNT using nonlocal Timoshenko beam theory and studied the influences of nonlocal parameter, length and radius of the SWCNTs and end supports on the nonlinear free vibration characteristics.

In this study, nonlinear buckling analysis of nonlocal boron nitride Timoshenko nano beam on elastic foundation based on MCST, nonlocal elasticity Eringen's model, and Von Karman nonlinear geometry theory are investigated. Finally, the effects of different parameters such as nonlocal Eringen's parameter, slenderness ratio, electric field, temperature changes and material length scale parameter on the nonlocal critical buckling load of Timoshenko nano beam are demonstrated.

2- Nonlocal Elasticity Theory

The well-known relation of nonlocal stress tensor (σ) in terms of the classical stress ($\sum$) is expressed as [1]:

$$(1-(e_0a)^2\nabla^2)\sigma_{ij}=C_{ijkl}\,\varepsilon_{kl} \qquad (1)$$

The boron nitride Timoshenko nano beam is modeled with radius r, length L and thickness h embedded in an elastic medium that is shown in Figure 1.

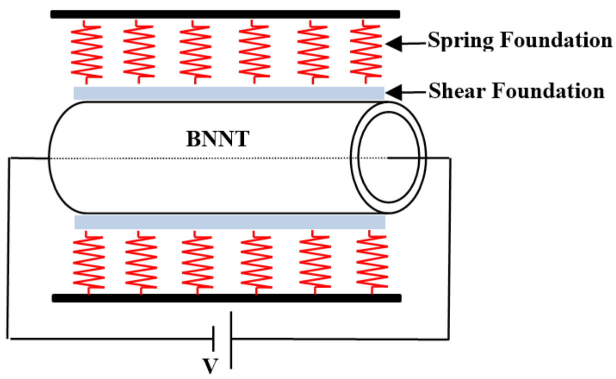

Figure 1. Schematic view of boron nitride Timoshenko nano beam

The displacements of an arbitrary point in the nano beam along the x- and z-axes, respectively, are [2]:

$$U(x,z,t)=U(x,t)+z\,\Psi(x,t)$$
$$W(x,z,t)=W(x,t) \qquad (2)$$

The von Karman type nonlinear strain-displacement relations are given by

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$$\varepsilon_{xx} = \partial U/\partial x + 1/2\left(\partial U/\partial x\right)^2 + z\left(\partial \Psi/\partial x\right)$$
$$\gamma_{xz} = \partial W/\partial x + \Psi \tag{3}$$

3- Governing Equations

Assuming potential energy U, kinetic energy T and the external work V due to surrounding elastic medium and buckling load, the Hamilton's principle is defined as:

$$\int_{t_1}^{t_2} \delta\Pi dt = 0$$
$$\delta\Pi = \delta T - \delta U - \delta V = 0 \tag{4}$$

where

$$U = 1/2\int_0^L \int_A \left(\sigma_{xx}\varepsilon_{xx} + \tau_{xz}\gamma_{xz} + 2\chi_{xy}m_{xy}\right)dA.dx$$
$$T = 1/2\int_0^L \left[\rho A\left(\partial U/\partial t\right)^2 + \rho A\left(\partial W/\partial t\right)^2 + \rho I\left(\partial \Psi/\partial t\right)^2\right]dx \tag{5}$$
$$V = -1/2\int_0^L \left(-k_w W + G_p\nabla^2 W - \bar{P}\nabla^2 W\right)W\,dx$$

where ε_{xx}, γ_{xz} are the axial and shear strain, respectively. The m_{xy} and χ_{xy} are defined as follows:

$$\chi_{xy} = 1/4\left[\partial\Psi/\partial x - \partial^2 W/\partial x^2\right], m_{xy} = 2Gl_m^{\,2}\chi_{xy} \tag{6}$$

where l_m is material length scale parameter. The nonlocal constitutive Eq. 1 can be approximated to one-dimensional form as [2]:

$$\sigma_{xx} - \left(e_0 a\right)^2 \partial^2\sigma_{xx}/\partial x^2 = E\varepsilon_{xx} - E\alpha_x T - h_{11}E_x$$
$$\sigma_{xz} - \left(e_0 a\right)^2 \partial^2\sigma_{xx}/\partial z^2 = G\gamma_{xz} \tag{7}$$

where G and E are elastic coefficients, α_x, h_{11}, T and E_x are thermal expansion, piezoelectric coefficients, temperature change and electric field, respectively. The normal resultant force N_x, bending moment M_x and shear force Q_x are obtained as:

$$N_x = \int_A \sigma_{xx}\,dA$$
$$M_x = \int_A \sigma_{xx}\,z dA \tag{8}$$
$$Q_x = \int_A \sigma_{xz}\,dA$$

Calculating the variational form of Eq. 5 and setting the coefficients of δU, δW and $\delta\Psi$ to zero, yield the following equations of motion:

$$N_{x,x} = \rho A U_{,tt}$$
$$\left(N_x W_{,x}\right)_{,x} + Q_{x,x} + \beta\left(\Psi_{,xxx} - W_{,xxxx}\right)$$
$$-k_w W + G_p W_{,xx} - \bar{P}W_{,xx} = \rho A W_{,tt} \tag{9}$$
$$M_{x,x} - Q_x + \beta\left(\Psi_{,xx} - W_{,xxx}\right) = \rho I\Psi_{,tt}$$

where $\beta = Gl_m^{\,2}A/2$. The relations between the stress resultants in local theory and nonlocal theory are defined as Eq. 10, where k_s is shear correction factor depends on the shape of the cross-sectional area of the nanobeam. Deriving N_x, M_x and Q_x from Eq. 9 and substituting into Eq. 10, the governing equations of motion in terms of displacements are obtained.

$$N_x - \left(e_0 a\right)^2 N_{x,xx} = EA\left[U_{,x} + 1/2W_{,x}^{\;2}\right]$$
$$-EA\alpha_x T - h_{11}AE_x$$
$$M_x - \left(e_0 a\right)^2 M_{x,xx} = EI\Psi_{,x} \tag{10}$$
$$Q_x - \left(e_0 a\right)^2 Q_{x,xx} = k_s GA\left(W_{,x} + \Psi\right)$$

4- Results and Discussion

In order to solve nonlinear governing equations of motion and corresponding boundary conditions, the Differential Quadrature (DQ) method is used to determine the critical buckling load of nonlocal boron nitride nano beam. To validate the obtained results of present work, comparisons have been carried out with the results of Murmu and Pradhan [3], showing good accuracy.

Figures 2 illustrates the effect of nonlocal parameter on the critical buckling load of nano beam. It is seen that increasing e0a leads to reduction of critical buckling load so that reduction rate for clamped-simply supported nano beam is more than simply-simply supported boundary condition.

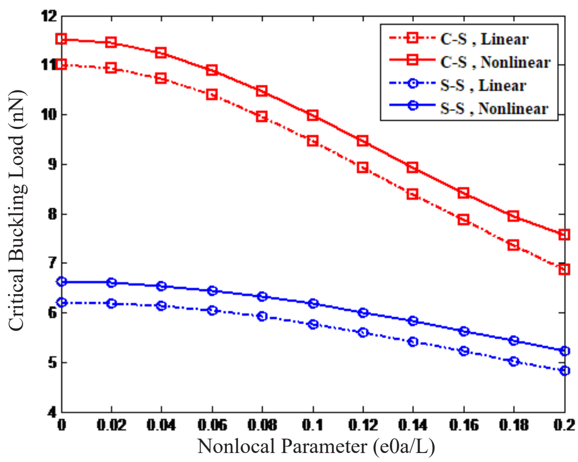

Figure 2. The effect of nonlocal parameter on the critical buckling load.

The influence of slenderness ratio on the critical buckling load is showed in Figure 3. According to this figure, as the L/r increases, the difference value between linear and nonlinear cases declined and closed to each other.

5- Conclusions

In this article, the nonlinear buckling analysis of nonlocal boron nitride Timoshenko nano beam on elastic foundation based on MCST, nonlocal elasticity Eringen's model, and Von Karman nonlinear geometry theory are investigated. The main results of this research can be summarized as follows:

1. Increasing the nonlocal parameter and slenderness ratio reduces stiffness and the critical buckling load. This reduction for simply-simply supported nano beam is less than clamped-simply supported ones. Also for large slenderness ratio, nonlinear analysis can be approximated by linear analysis where this behavior can be observed with difference reduction between linear and nonlinear

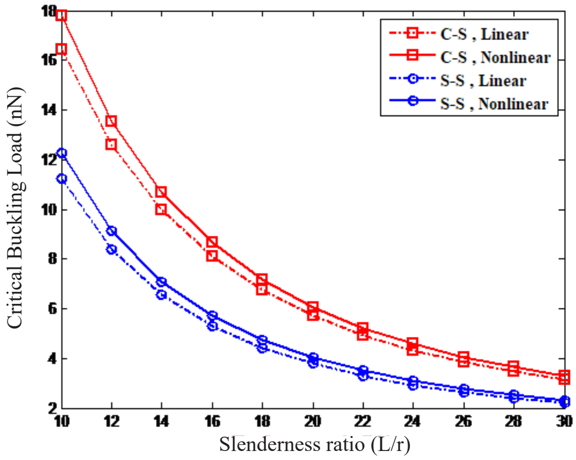

Figure 3. The effect of slenderness ratio on the critical buckling load.

analysis with increasing slenderness ratio.

2. The results show that by increasing electric field and temperature change, the critical buckling load is declined partially. This is due to the reduction of nano beam stiffness caused by applied electrical voltage and temperature changes and thus reduces the critical buckling load.
3. By enlarging material length scale parameter, critical buckling load is increased so that the rate of enhancement for clamped-simply supported nano beam is more than simply-simply.

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