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Thermoelastic Damping Effect Analysis in Micro/Nano Flexural Resonators

M. Moosapour[1], M.A. Hajabasi[2]*

1- Department of Mechanical Engineering, The University of Guilan, Rasht, Iran
2- Department of Mechanical Engineering, Shahid Bahonar University of Kerman, Kerman, Iran

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ABSTRACT

Understanding effects of thermoelastic damping on the vibration parameters such as natural frequency and frequency-sensitive is essential for the design of micro-nano-electromechanical systems. In this paper, the effects of thermoelastic damping in micro- and nanomechanical resonators beam with a rectangular cross section is analyzed. Governing equations in present system are coupled of heat conduction equation and equation of motion where this methodology has been applied to three- dimensional analysis. To solve the governing equations analytically with considering suitable assumptions, first the coupled heat conduction equation is solved for the thermoelastic temperature field by considering three-dimensional heat conduction along the length, width and thickness of the beam.Next, thermoelastic coupling is modeled into the equation of motion for flexural vibrations through a temperature-dependent moment of temperature distribution. Frequency shifts and quality factor due to thermoelastic damping are analyzed. For special cases, the obtained frequency shift is compared with the result of the frequency shifts computed using 2D heat conduction; in addition, the obtained quality factor is compared with exact 1D heat conduction. The results obtained showed that the model presented in this paper are in good agreement with the other models and it predicted the effects of thermoelastic damping on the behavior of micro-nano resonators accurately.

KEYWORDS:

Thermoelastic Damping, Micro- nano Resonator, Quality Factor, Frequency Shift.

* Corresponding Author, Email: hajabasi@uk.ac.ir

1- Introduction

The TED represents loss in energy from an entropy rise caused by the coupling between heat transfer and strain rate. In the 1937's, Zener was the first to realize that TED may be a significant dissipation mechanism in mechanical resonators [1,2]. Lifshitz and Roukes [3] developed thermoelastic equations of a vibrating beam. Prabhakar et al. [4] presented an exact 2D analysis of frequency shifts due to thermoelastic coupling for flexural vibrations of a beam. However most studies of TED have been based on analytical models, but those are subject to very restrictive assumptions so that they are not sufficiently accurate to predict the behavior of complex (3D) structures.

The present work focuses on the effects of TED in a thin beam resonator undergoing flexural vibrations. In order to accurately determine TED effects, the coupled thermal conduction equation is solved for the temperature field by considering (3D) heat conduction along the length, width and thickness of the beam.

2- Methodology, Discussion and Results

2-1- Equations of Motion with 3D Heat Honduction for Vibrations of a Thermoelastic Beam

Consider a thin beam resonator with length L, width D and uniform thickness h. The equation of motion for flexural vibrations of the beam with 3D heat conduction equation can be obtained as

$$\frac{EI_y}{1-\nu^2}\frac{\partial^4 w_0}{\partial x^4}-\frac{8E^2\alpha^2T_0}{(1-\nu)^2\rho c_v}\frac{1}{DLh}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\left(R_{mpq}+iI_{mpq}\right)e^{i\omega_n t}+\rho A\frac{\partial^2 w_0}{\partial t^2}=0 \tag{1}$$

where

$$R_{mpq}=\frac{\alpha_m^2\omega_n^2}{\omega_n^2+\dfrac{k^2}{\rho^2c_v^2}\left(\alpha_{mm}^{\ 2}+\beta_p^2+\gamma_q^2\right)^2}\lambda\chi^2\sin(\alpha_m x) \tag{2.a}$$

$$I_{mpq}=\frac{\alpha_m^2\omega_n\dfrac{k}{\rho c_v}\left(\alpha_m^2+\beta_p^2+\gamma_q^2\right)}{\omega_n^2+\dfrac{k^2}{\rho^2c_v^2}\left(\alpha_m^2+\beta_p^2+\gamma_q^2\right)^2}\lambda\chi^2\sin(\alpha_m x) \tag{2.b}$$

2-2- Solution of the Equation of Motion for a Thin Beam under Thermoelastic Damping Effects

The Galerkin weighted residual technique yields a nonlinear algebraic equation in ω_n, as follows

$$\omega_n^2=\omega_n\big|_{isothermal}^2-\frac{8E^2\alpha^2T_0}{(1-\nu)^2\rho c_p}\frac{1}{D^2L^2h^2}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\int_0^L\left(R_{mpq}+iI_{mpq}\right)\phi_n(x)\,dx \tag{3}$$

Next, Eq. (3) is solved for ω_n using an iterative procedure. Then the iterative process is terminated, and the dimensionless fractional frequency shift due to thermoelastic coupling is calculated as

$$\Delta_\omega=\frac{\mathrm{Re}(\omega_n^{(K^*)})-\omega_n\big|_{isothermal}}{\omega_n\big|_{isothermal}} \tag{4}$$

Consequently, in terms of the derived complex frequency, the quality factor related to termoelastic damping can be expressed in terms of the imaginary and real parts of the frequency. The quality factor, which is the fraction of energy lost per cycle of vibration, is given by

$$Q=\frac{\sqrt{\mathrm{Re}^2(\omega_n^{(K^*)})+\mathrm{Im}^2(\omega_n^{(K^*)})}}{2\left|\mathrm{Im}(\omega_n^{(K^*)})\right|} \tag{5}$$

Where the factor 2 arises from the fact that the mechanical energy of the beam is proportional to the square of the amplitude of deformation [3].The imaginary part of the resonant frequency is small compared to the real part. Therefore, the thermoelastic quality factor can be approximated by the following expression

$$Q\approx\frac{1}{2}\left|\frac{\mathrm{Re}(\omega_n^{(K^*)})}{\mathrm{Im}(\omega_n^{(K^*)})}\right| \tag{6}$$

2-3- Numerical Results and Discussion

2-3-1- Frequency Shifts

To validate the solution procedure, the results of 3D theory of frequency shifts due to thermoelastic coupling are compared with the 2D exact results of Prabhakar et al [4]. It can be seen in Figure 1. This results are calculated for the flexural vibration of doubly- clamped single-crystal silicon beam with thickness ranging from 3 to 10 µm, a fixed length of L=90 µm, the width-to-thickness ratio is fixed at D/h=5 and ν=0 (to approximate the case of a simple beam). According to Figure 1, at small thicknesses, a difference of 10% can be detected between 2D and 3D theory of frequency shifts due to thermoelastic coupling. However, the differences between the two theories increase as thickness of the beam increases. There is a 27% difference between the two theories for beams with thickness h=10 μm.

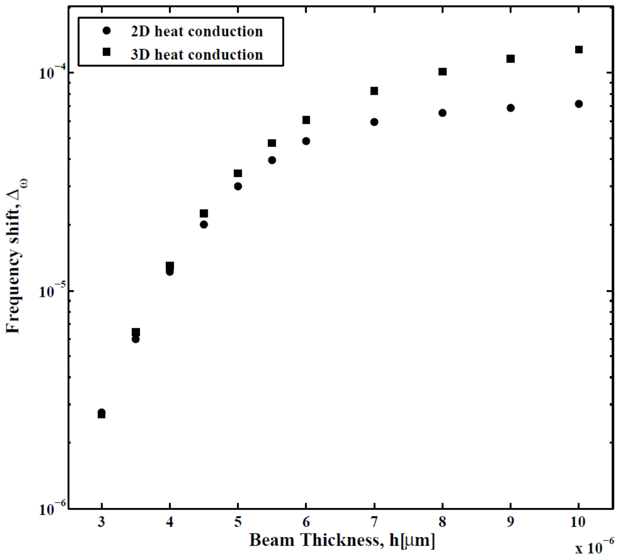

Figure 1. Comparison of frequency shifts computed using 3D theory of frequency shifts with the 2D exact results of Prabhakar et al [4].

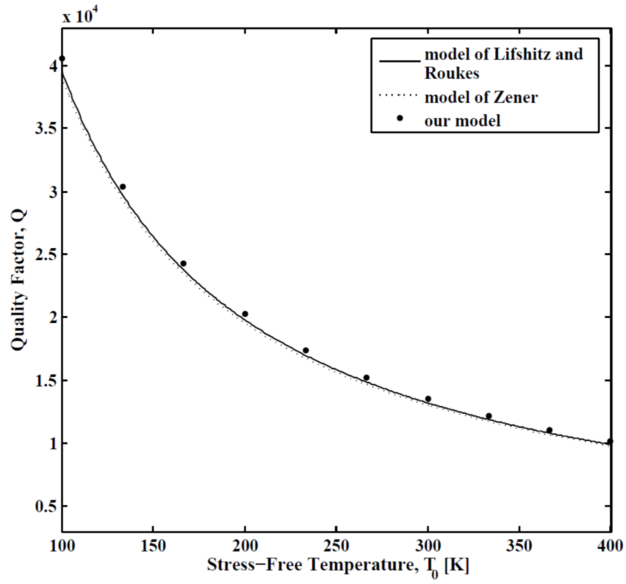

Figure 2. Comparison of the quality factors, Q, obtained using our model to those of Lifshitz and Roukes [3] and Zener [1,2] for various values of the stress-free temperature T_0.

2-3-1- Quality Factor

Consider a beam with h=5μm, D/h=5, L/h=10, v=0 and stress-free temperature ranging from 100 to 400 K. Figure 2 shows the variations of quality factor, Q, with stress-free temperature T_0, for cantilevered beams. In this figure similar results in the works of Lifshitz and Roukes [3] and Zener [1,2] are also presented for comparison. As illustrated in this figure, the results of Eq. (6) are in good agreement with the results of exact 1D theory of Lifshitz and Roukes [3] and Zener [1,2].

3- Conclusions

In this paper, quality factor and frequency shift for flexural vibration of thin beam resonators are analyzed. In order to accurately determine TED effects, the coupled thermal conduction equation is solved for the temperature field by considering 3D heat conduction along the length, width and thickness of the beam. The theoretical framework closely follows the exact 2D theory developed by Prabhakar et al. [4] to compute the effects of thermoelastic coupling on frequency shifts. The Galerkin weighted residual technique is used to obtain frequency shift and thereupon the quality factor thin beam resonator.

The main conclusions can be classified as follows:

- At small thicknesses a difference of 10% can be detected between 2D and 3D theory of frequency shifts due to thermoelastic coupling. However, the differences between the two theories increase as thickness of the beam increases. There is a 27% difference between the two theories for beams with thickness h=10 μm.
- The quality factor, Q, decreases as the stress-free temperature, T_0, increases.

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