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Analysis of Sides Ratio Effect on Propagation of Ultrasonic Guided Waves in a Bar with Rectangular Section

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ABSTRACT

In this research, sides ratio effect of ultrasonic guided waves in a bar with rectangular section is investigated and wave structure is calculated. Motion of ultrasonic guided waves in the mentioned bar is considered as a three-dimensional problem. In homogeneous system of equations that is obtained to satisfy stress free surface boundary conditions, coefficients determinant of this system must be equal to zero. So, frequency equation is obtained. The real roots of this equation are extracted and frequency spectrum, phase and group velocity diagrams and wave structure for longitudinal, torsional and bending waves are plotted by writing a computer program. It is observed that by increasing the bar section sides ratio, when the frequency increases, the longitudinal and torsional waves phase velocity decreases and bending waves phase velocity increases. In addition, for some modes, replacing particles on the bar surface is considerable and for some others is negligible.

KEYWORDS:

Ultrasonic guided waves, Phase velocity, Group velocity, Wave structure, Rectangular section.

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1- Introduction

Ultrasonic guided waves have several applications in different industries and they have been considered as a method to diagnose various defects. Many researches have been conducted on the wave propagation in plates and cylinders, but still there is no exact solution for many other sections. The reason behind this limitation is that two dimensions in plates can be considered infinite and in cylinder properties of axial symmetry can be used to simplify equations; but in rectangular sections, the problem is complicated. Effects of lateral motion components on the longitudinal components, transforms the problem to a three dimensional problem; therefore, solving the governing equations, the simplifying and adequate approximates must be used [1].

In this study, the superposition principle is used for calculation of displacement vector and extraction of frequency equations of longitudinal, bending and torsional waves. For doing this, by writing a computer program, roots of the frequency equation was extracted and frequency spectrum, phase and group velocity and wave structure was plotted for a bar with rectangular section and the width of 2a and thickness of 2b. Suggestions are given for improving the procedures of ultrasound tests by selecting the appropriate mode. In addition, it is tried to provide the possibility of studying other sections.

2- Solving the equations

The equation of motion in an elastic isotropic environment is presented as follows:

$$(\lambda+\mu)\nabla(\nabla\cdot\vec{u})+\mu\nabla^2\vec{u}=\rho\frac{\partial^2\vec{u}}{\partial t^2} \tag{1}$$

Where λ and μ are the Lame constants, u the displacement vector, ρ density, t time, and ∇ and ∇^2 are the Gradient and Laplacian functions. The displacement vector is written by using the Helmholtz analysis method as a combination of gradient a scalar potential function φ and curl of a vector potential function H that its divergence equals zero.

By replacing the displacement vector in Equation (1), the following equations are obtained.

$$\nabla^2\varphi=\frac{1}{c_L^2}\frac{\partial^2\varphi}{\partial t^2} \tag{2}$$

$$\nabla^2\overline{H}=\frac{1}{c_T^2}\frac{\partial^2\overline{H}}{\partial t^2} \tag{3}$$

$$c_L^2=\frac{\lambda+2\mu}{\rho} \quad , \quad c_T^2=\frac{\mu}{\rho} \tag{4}$$

Using the method of superposition principle and choosing appropriate trigonometric functions, the potential functions are obtained by undetermined coefficients, and after calculation of components of the displacement, the equations of motion is obtained. By applying the equations of displacement-strain and strain-stress, components of stress are calculated.

By applying the boundary conditions that the external surfaces are stress free, the homogeneous system of equations is obtained and it can be used to calculate the undetermined coefficients. By performing long and complex mathematical operations, the homogeneous system of equations is extracted for each of the longitudinal, bending and torsional waves. In this equations, the undetermined coefficients appear as A and B that are unlimited in the number. For the system of homogeneous equations to have answers, the determinant of its coefficients must all equal to zero. Therefore, the frequency equation of the propagable waves in the bar is obtained that the real roots of the equations are extracted with four unknown coefficients. This equation is only solvable with presence of the two variables of frequency and wave number. For each given wave number, numerous frequencies are obtained that show the propagable modes and they are numbered from the first obtained wave, respectively.

The answer of the frequency equation is only dependent on the Poisson ratio and the sides ratio, because, in the frequency equations, the ratios of phase and group velocity and the ratio of the Lame coefficients are only dependent on the Poisson ratio. Therefore, making these parameters dimensionless would be helpful in better provision of the results. The dimensionless parameters are obtained from Equation (5).

$$\begin{gathered}\bar{\omega}=\frac{\omega b}{c_T} \quad , \quad \bar{c}_p=\frac{c_p}{c_T} \quad , \quad \bar{c}_g=\frac{c_g}{c_T} \quad , \quad \bar{k}=kb \\ \bar{u}_i=\frac{u_i}{u_{i_{\max}}} \quad , \quad \bar{y}=\frac{y}{b} \quad , \quad \bar{x}=\frac{x}{a}\end{gathered} \tag{5}$$

3- Results

The graphs of the frequency spectrum and phase velocity for the first longitudinal mode of the rectangular bar with the Poisson ratio of 0.3 and the different sides ratios are presented in Figures 1 and 2. By increasing the ratio of width to thickness, the frequency of propagable waves is decreased especially for the small wave numbers and the phase velocity tends faster towards a constant value (the

Rayleigh wave velocity).

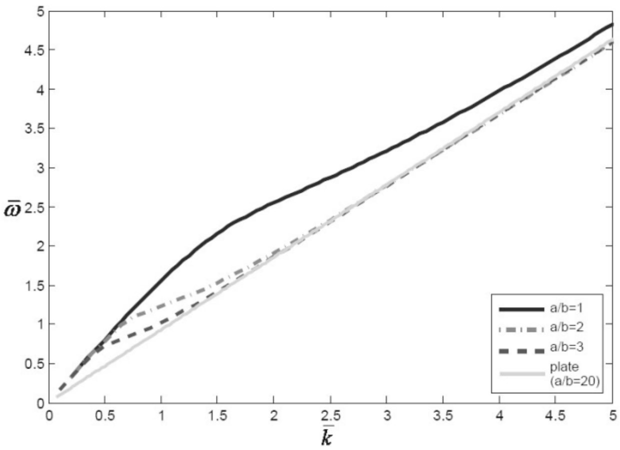

Figure 1: The frequency spectrum of the first longitudinal mode for the bar with rectangular section

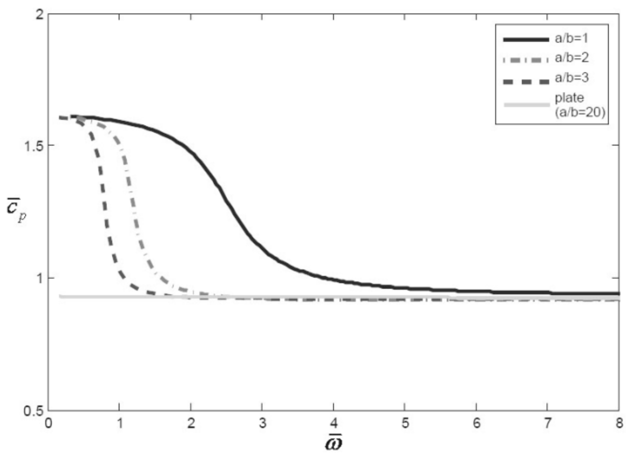

Figure 2: The phase velocity of the first longitudinal mode for the bar with rectangular section

The diagrams for the frequency spectrum and phase velocity for the first six longitudinal modes in the bar with sides ratio of 2 are presented in the Figures 3 and 4. In Figure 3 it is observed that the frequency of the first mode is tending toward the zero in k=0, but the other modes have a cutoff frequency that is not propagated in the lower frequencies. In Figure 4 it is observed that the phase velocity diagrams are tending to a constant value that is the Rayleigh wave velocity.

4- Conclusions

The superposition principle method was used for studying the waves propagation in a bar with rectangular section. This method leads to determination of the eigenvalues of a homogeneous system of equations with infinite equations. The obtained results are in a good agreement with results of Tanaka and Iwahashi study [2] which are obtained by the same method for a square section. According

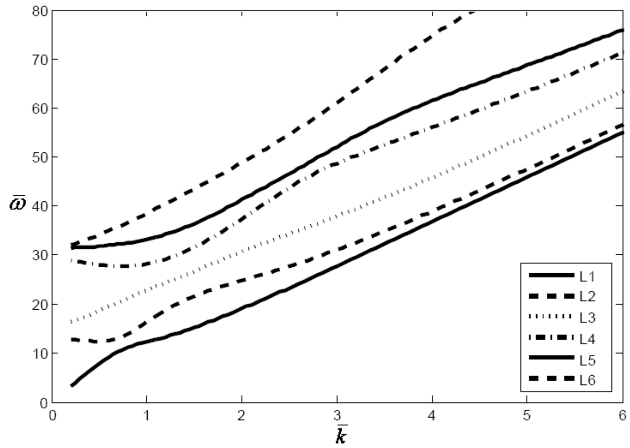

Figure 3: The frequency spectrum of the longitudinal waves in the bar with rectangular section

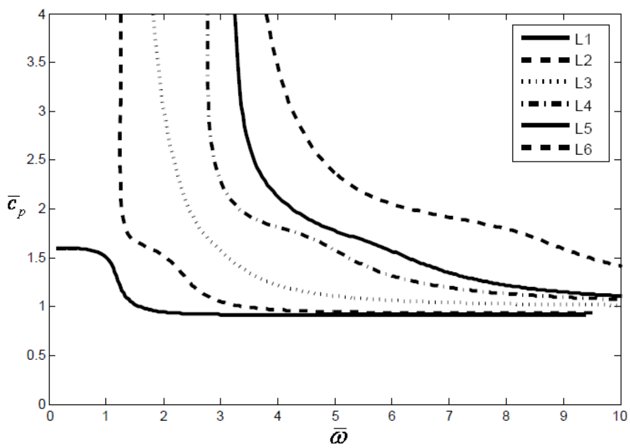

Figure 4: The phase velocity of the longitudinal waves in the bar with rectangular section

to the results, it is shown for the large values of wave number, the phase velocity of the first longitudinal mode, regardless of the size of the section dimensions, tends to a constant number that is the Rayleigh wave velocity. For the small values of the wave number, with increasing the sides ratio, the phase velocity of all modes except the first torsional mode, tends faster toward this constant number. In a bar with rectangular section, except the first longitudinal and bending modes, other modes have a cutoff frequency. The cutoff frequency is lower in a bar with rectangular section than the cutoff frequency in the square section and with increasing the sides ratio, the cutoff frequency reduces more.

5- References

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