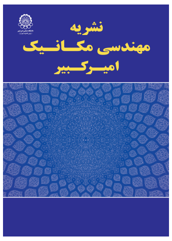

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Reconstruction of Gappy Unsteady Flow Fields Using Improved Reduced Order POD Model Based on Temporal Decomposition Procedure

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ABSTRACT: In this paper, the Proper Orthogonal Decomposition (POD) method has been used for gappy data reconstruction of the unsteady incompressible viscous flow and calculation of related forces such as aerodynamics coefficients. The present approach is based on combination of POD method and solution of an optimization problem to get a reduced order model. This model reconstructs missed data or incomplete snapshots with relevant accuracy and high speed. Two kinds of models have been used in this manner. In the first approach, a simple iterative method is used to reconstruct the relative missed or incomplete basis functions of snapshots ensemble. The Second method, which is proposed recently by the author, uses a time advancing and domain decomposition method along the time direction. It reconstructs the incomplete ensemble members with high missing percentage. The essential trait of this method is on application of step-by-step reconstruction of primary ensemble. It reduces the number of unknown variables to obtain an accurate model to recover the missed data. The outcome results of the two approaches have been compared with direct simulation data which show appropriate ability and good agreement.

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1- Introduction

In general, the analysis methods of fluid mechanics problems contain experimental and computational approaches and their characteristics mentioned in different references [1].
Measuring devices limitation for data acquisition in different flow field points and dependency of numerical models to computer hardwares in order to increase the number of computational nodes are the most notable features of fluid mechanics analysis methods. Developing methods, which can estimate and compute more information from flow field points such as data acquisition accuracy, lack of relevant data (related to laboratory methods) and reducing the number of calculations to reduce the cost and time (corresponding to computational methods), can be considered as a suitable solution. Application of Proper Orthogonal Decomposition (POD) which has the ability to extract energetic structures of flow field and thus can reduce number of calculations with maintain the quality and accuracy of them, may be as an appropriate approach. Lumley suggested that the POD method can be used to extract the coherent structure of turbulent flow [2]. By introducing snapshots method by Sirovich, POD is considered as an efficient tool for developing reduced order models of complex dynamical systems which have relevant data from experiments or direct numerical solutions [3]. Taeibi et al. used this method to calculate the aerodynamics coefficients and analysis of compressible flow field [4]. Wilcox used this method to find the position of sensors in order to control vortex shedding in downstream of a cylinder [5]. Sabetghadam et al. investigated about application of this method to reconstruct missed data or incomplete snapshots ensemble of the unsteady flow field around a square cylinder [6].

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The aim of this study is about application of this method to reconstruct the missing data of the unsteady flow field. For this purpose, using POD-Snapshots method, a minimization approach used for calculating the related coefficients of the missed points.

2- Proper Orthogonal Decomposition

Lumley proposed a definition from coherent structures of turbulent flows as a function of spatial variables that have the highest energy level. Thus, these coherent structures maximize the following expression:

$$\frac{\left(u(x,t),\varphi(x)\right)^2}{\left(\varphi(x),\varphi(x)\right)} \tag{1}$$

If $\varphi(x)$ maximizes Equation 1, it can be concluded that if flow field is projected along $\varphi(x)$ its mean energy is more than projection of field along any other basis functions. Eigenfunctions (POD modes) have been calculated using Sirovich snapshots method. POD Modes are eigenfunctions of the following matrix:

$$K(x,x') = u_k(x,t), u^*(x',t^k) \tag{2}$$

In this study, the singular value decomposition method has been used to solve the related eigenvalues problem. Now with these modes the flow field can be reconstructed by the following equation:

$$u(\vec{x},t) = \sum_{i=1}^{M} a^i(t)\varphi^i(\vec{x}) \tag{3}$$

3- Reconstruction of Flow Field Data

In this section discussed about application of POD in the reconstruction of the flow field around the body. Three methods used to estimate the flow field around the body using POD as follows[4]:

- POD with Interpolation,
- Gappy-POD methods for Missed Data Reconstrcution,
- Iterative Gappy-POD method for Incomplete Ensemble.

The methods, which are used in this study, contain two types. In the first method, the least square approach is used to solve minimization problem based on POD modes and using an iterative method to reconstruct incomplete snapshots ensemble. The second method has the same structure with the first method. This method uses a time advancing reconstruction method and completes the initial snapshots ensemble to reconstruct of incomplete basis.

4- Iterative Gappy-POD Method with Time Advancing

Gappy iterative method has a problem when the number of stations (time-dependent snapshots) with missed data is large. The method proposed in this study can greatly solve this problem. Then snapshots ensemble is generated locally and with new structure and member. If the initial observation matrix is assumed as follows:

$$\mathbf{u} = \left\{ u_1 \quad \ddot{u}_2 \quad \ddot{u}_3 \quad \ldots\ldots \quad u_n \right\} \tag{4}$$

Where, u_1, u_4 and u_n are members of primary ensemble with complete information and $\ddot{u}_2$ and $\ddot{u}_3$ are members with less and more missed information respectively.

Therefore, primary matrix will be included new block matrices where the number of their arrays contains missed data will be less. The new matrix for step by step reconstruction of missed arrays will be as follows:

$$\tilde{\mathbf{u}} = \left\{ \underbrace{\left[\underbrace{u_1 \quad \ddot{u}_3 \quad u_4} \right]}_{New\ local\ Ensemble} \quad \ldots\ldots\ldots \right\} \tag{5}$$

Then, the missing member is reconstructed by method described in the previous sections. This member is added to initial ensemble as a complete member at any time step.

5- Mode Selection and Reduced Order Model Construction

It is clear in a modal solution process by increasing the number mode, the result will be more accurate compared to exact solution. If the POD modes are arranged based on their energy, this point will clearly see that by using a smaller number of modes, a high level of kinetic energy of flow field can be captured. This means that by described method a reduced order model can be obtained. The number of modes, which are captured a higher level of energy, is calculated by the following equation:

$$\kappa = \frac{\sum_{i=1}^{N_r} \lambda_i}{\sum_{i=1}^{N_{total}} \lambda_i} \tag{6}$$

Where λ is eigenvalue of snapshots ensemble and N_r is the required number of modes for reconstruction of reduced order model.

6- Results and Discussion

Results are presented for unsteady incompressible viscous flow around a square cylinder at Reynolds number of 100. An ensemble with 38 members from solution of the flow field at different time steps has been considered as an input. In Figures 1 and 2, four strongest modes of the streamwise velocity and pressure are shown, respectively.

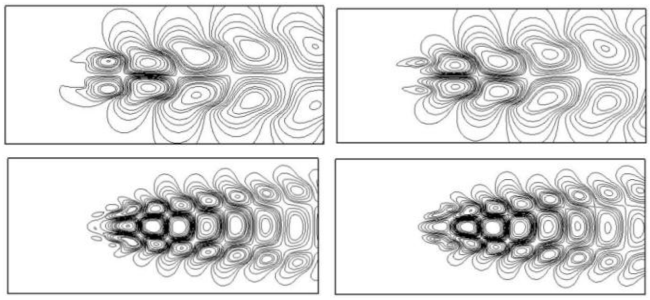

Figure 1. Four strongest modes of streamwise velocity component

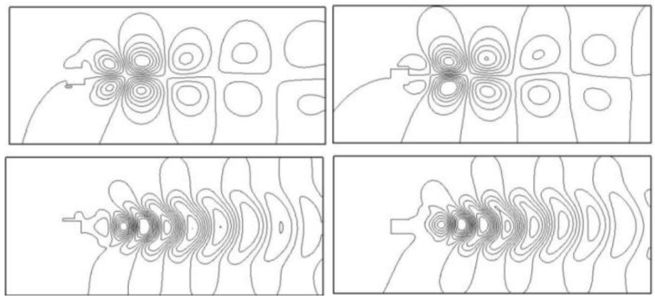

Figure 2. Four strongest modes of pressure field

In Fig. 3, contours of streamwise velocity component are shown for the original data (obtained from direct numerical solution), flow field with 90% missing data, reconstructed flow field by standard repairing approach and reconstructed field with iterative time advancing method at $t = 4.38$.

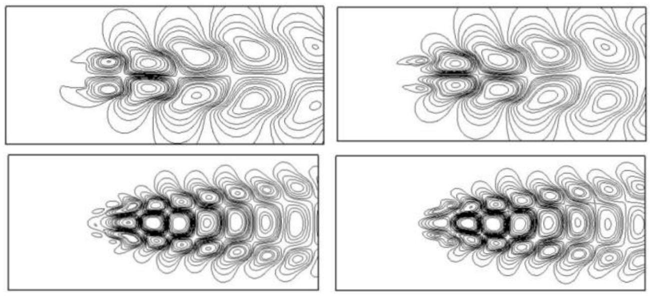

Figure 3. Streamwise velocity contour for missing data with 90%(left-top), original snapshot (right-top), reconstructed using simple iterative method (left-bottom) and reconstructed using time advancing iterative method (right-bottom)

7- Conclusion

In this study application of POD method for reconstruction of unsteady flow fields with a high percentage of missing data is discussed. According to the results, it can be said that the proposed method has good and right capability for reconstructing flow field with high percentage of missed data. The reason for this is that the primary snapshots ensemble,

including missing data, is improved at each time step as a time advancing approach with new calculated data.
Eventually, it can be said with respect to the proposed model, the method could be simply used for other applications such as image processing, reconstruction of incomplete data of laboratory tests and data acquisition for meteorological and oceanographic studies

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