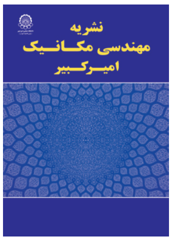

Amirkabir Journal of Mechanical Engineering

Amirkabir Journal of Mechanical Engineering, 49(4) (2018) 279-282
DOI: 10.22060/mej.2016.724

Generalized Predictive Filter for Discrete-Time Linear Systems

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ABSTRACT: In this paper, based on the duality between the predictive control and general estimation problem, two new predictive filters, named generalized predictive filter and generalized predictive Kalman filter, are developed. The major advantage of the new filters over the existing predictive filters are that their structure are very simple and their application as a recursive filter is not complicated. Unlike the Kalman filter, these proposed predictive filters assume that process noise and model error are not equivalent and there are no limitations about the form of model error so that this model error can appear in a nonlinear form or even a colored noise. By minimizing a quadratic cost function consisting of a measurement residual term and a model error term respect to the process model error, the optimal model error is determined. Compensation of this model error in the time update state model provides accurate estimates even in the presence of dynamic uncertainty. Combination of Kalman filter and generalized predictive filter improves the performance and robustness of Karman filter. The validity of the suggested filters is illustrated by a numerical example and their performance and robustness are compared with those of KF and the fading Kalman filter.

Review History:

Received: 7 November 2015
Revised: 3 March 2016
Accepted: 9 May 2016
Available Online: 11 August 2016

Keywords:

Generalized Predictive Filter
State Estimation
Kalman Filter
Optimal estimation
Uncertainty

1- Introduction

In general, the predictive filters based on minimization of a cost function including both of the measurement residual term and the model error term with respect to the model error, estimate the optimal model error on line and use this estimation to correct the estimated states in each time step [1].

In this paper, based on concepts of the generalized predictive control (GPC) introduced in [2, 3], two new filters called the generalized predictive filter (GPF) and the generalized predictive Kalman filter (GPKF) for discrete-time linear systems are presented. GPF and GPKF have a prominent advantage over the predictive filters presented in [1, 4-7]. In fact, the existing predictive filters due to the use of Lie derivatives in their formulation have very complex mathematical algorithms. This complexity makes it difficult to understand the algorithm and makes the use of predictive filters be accompanied with great difficulties. This drawback seriously curtails their widespread utilization and causes their features to remain unexplored. Therefore, in order to provide some facilities available to engineers, it is necessary to propose an alternative and easy approach to achieve the predictive filter. This research overcomes this problem by introducing GPF and GPKF as two effective and facile PFs. The derivation of these new filters is very simple and implementing them as a recursive filter is very straightforward.

2- Development of Generalized Predictive Filters

The state space and measurement equations of the discrete-time linear system are:

$$x_{k+1} = F_k x_k + G_k^w w_k \tag{1}$$

$$\tilde{y}_k = H_k x_k + v_k \tag{2}$$

where $x \in R^{n_x}$, $\tilde{y} \in R^{n_y}$, and k denote the true state vector, the measurement vector and the time step, respectively. F_k, G_k^w, and H_k represent the system matrix, the noise distributer matrix and the measurement matrix, respectively. Also $w_k \in R^{n_x}$ and $v_k \in R^{n_y}$ are the random process noise and measurement noise, respectively, with zero-mean and Gaussian white-noise distributed process. In order to develop the GPF, the state and output estimation of the system with Eq. (1) and (2) are assumed as follows:

$$\hat{x}_{k+1} = F_k \hat{x}_k + G_k^d \hat{d}_k \tag{3}$$

$$\hat{y}_k = H_k \hat{x}_k \tag{4}$$

where $\hat{x}_k \in R^{n_x}$ and $\hat{y}_k \in R^{n_y}$ denote the state and output estimation vector, respectively, $\hat{d}_k \in R^{n_d}$ is the model error vector, and G_k^d represents the model-error distributer matrix. Using Eq. (3) and (4), one step ahead of the system output will be as follows:

$$\hat{y}_{k+1} = H_x \hat{x}_k + V \hat{d}_k \tag{5}$$

where $V = H_{k+1} G_k^d$ and $H_x = H_{k+1} F_k$. To achieve the optimal estimate of the model error, $\hat{d}_k$, the cost function is defined as:

$$J = \left(\tilde{y}_{k+1} - \hat{y}_{k+1}\right)^T R^{-1} \left(\tilde{y}_{k+1} - \hat{y}_{k+1}\right) + \hat{d}_k^{\,T} W \hat{d}_k \tag{6}$$

By minimizing the cost function with respect to $\hat{d}_k$, the optimal solution for the model error can be obtained at any time step:

$$\hat{d}_k = K_{GPF}\left[\tilde{y}_{k+1} - H_x \hat{x}_k\right] \tag{7}$$

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where $K_{GPF}=(V^T R^{-1} V+W)^{-1} V^T R^{-1}$.
After obtaining the optimal model error, the GPF can be applied as a recursive filter in accordance with two steps: first, the optimal model error is computed from Eq. (7). Second, the estimated states are corrected using this model error through Eq. (3).

3- Development of Generalized Predictive Kalman Filter

GPKF is a combination of KF and GPF. In the Kalman filter's equations for the systems with equations (1), (2), if the time-update equation of the priori estimated states, $\hat{x}^-_{k+1}=F_k \hat{x}^+_k$ (taken from [8]), is replaced by the following relationship, the generalized predictive Kalman filter is created:

$$\hat{x}^-_{k+1} = F_k \hat{x}^+_k + G^d_k \hat{d}_k \tag{8}$$

The model error vector $\hat{d}_k$ is obtained for each time step during the optimization process through Eq. (7).

4- Simulation Results and Discussion

To evaluate the performance of the newly developed filters, numerical simulations are performed and new filters are compared with two well-known filters, Kalman filter (KF) and fading Kalman filter (FKF), in term of robust performance (see [8,10] for details of these filters). The states estimation of a stochastic system from the turbofan engine F404 is the subject of study. In this example, this system is subjected to temporary and intense uncertainties [9]:

$$x_{k+1} = \begin{bmatrix} 0.9305+\delta_k & 0 & 0.1107 \\ 0.0077 & 0.9802+\delta_k & -0.0173 \\ 0.0142 & 0 & 0.8953+0.1\delta_k \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} w_k \tag{9}$$

$$y_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x_k + v_k \tag{10}$$

where δ_k is an uncertain model parameter as follows

$$\delta_k = \begin{cases} 0.1 & 50 \le k \le 100 \\ 0 & otherwise. \end{cases} \tag{11}$$

The system noise covariance Q and the measurement noise covariance R are equal to 0.05. The real initial state of the system is $\hat{x}_0=[1\ 1\ 1]^T$, but in simulation these real states are unknown and it is assumed that they are $\hat{x}_0=[3\ 3\ 3]^T$. To estimate the system states by GPF and GPKF, it is required to design an error. So, a two-dimensional error vector multiplied by a model-error distributer matrix with dimensions of 3×2 is selected as the error term and is added to the propagation model:

$$x_{k+1} = \begin{bmatrix} 0.9305+\delta_k & 0 & 0.1107 \\ 0.0077 & 0.9802+\delta_k & -0.0173 \\ 0.0142 & 0 & 0.8953+0.1\delta_k \end{bmatrix} x_k + \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \\ 0 & 0 \end{bmatrix} d_k + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} w_k \tag{12}$$

Also, the weight matrix W of GPF and GPKF is selected as $0.3I_{2*2}$. With these assumptions, the state estimates of the system have been simulated using GPF, GPKF, KF and FKF and the simulation results have been shown in Figs. 1 and 2. As depicted in these figures, the performance of the new predictive filters, is far better and far beyond than that of the Kalman filter and fading Kalman filter. In fact, although FKF estimates the states of the uncertain system more precisely than what KF does; however, its accuracy is very much lower than GPF's and GPKF's.

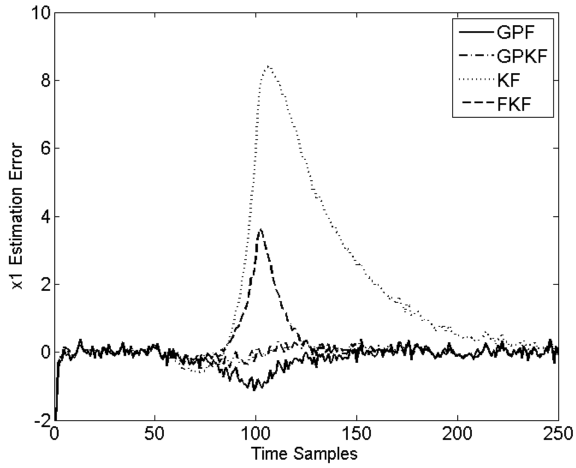

Figure 1. The x1 state estimation error

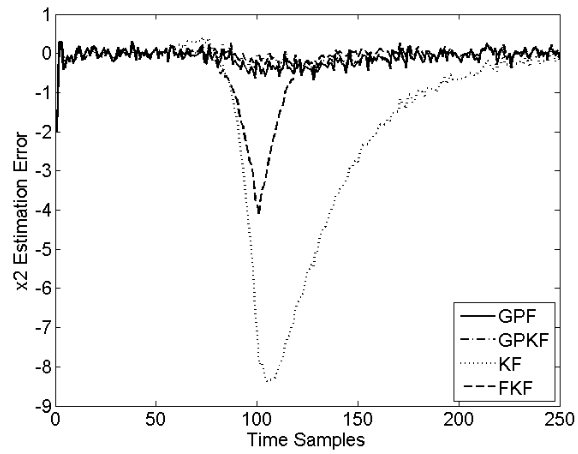

Figure 2. The x2 state estimation error

5- Conclusions

In this research, two new filters, state estimators, called the generalized predictive filter (GPF) and the generalized predictive Kalman filter (GPKF) were developed for discrete-time linear systems. The derivations of the GPF and GPKF are easier to understand than those of the existing predictive filters and applying them as a recursive filter is easily possible. Despite the simplicity of their structure, their robust performance against the model uncertainty is superior to the Kalman filter and fading Kalman filter. This advantage of GPF and GPKF is due to the implementation of mechanisms that estimate and compensate the model error optimally and more effectively and is due to having the finite-impulse-response structure as well. In fact, addition of this model compensation mechanism to Kalman filter makes this filter more accurate against the model uncertainty.

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Please cite this article using:

M. Fathi, N. Ghahramani, M.A. Shahi Ashtiani, M. Fallah, A. Mohammadi, Generalized Predictive Filter for Discrete-Time Linear Systems, *Amirkabir J. Mech. Eng.*, 49(4) (2018) 795-804.

DOI: 10.22060/mej.2016.724