Determination of Steady State Thermal Stress Intensity Factors for Semi-Elliptical Circumferential Cracks in Cylinders

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ABSTRACT: In this paper, the closed-form stress intensity factors are calculated at the deepest point of a circumferential semi-elliptical crack located at the inner surface of a cylinder. The cylinder is subjected to pressure (internal and external) and the inner surface of the cylinder is subjected to convection cooling. To solve the problem, initially, a weight function is derived for the deepest point of the circumferential semi-elliptical crack using two reference loads. Then, the steady state solution of the thermoelasticity problem is derived and, finally, the stress intensity factors are extracted using the weight function method. For some special cases of loading, the results of the present theory are compared with available solutions in the literature indicating an acceptable agreement. Moreover, the effects of crack relative depth and aspect ratio and heat transfer type on the thermal stress intensity factors are studied. The extracted results demonstrate that for some cases of loading and crack geometry, shallow cracks are more critical than deep ones and the solution of conduction heat transfer is more conservative than the forced convection one.

1- Introduction

Stress intensity factor is a parameter describing crack behavior; its calculation is necessary for predicting fatigue crack growth, stress corrosion cracking and sudden failure. Surface cracks are recognized as the most prevalent type among the others in pressure vessels used in different industries. One of the most common types of surface cracks is circumferential semi-elliptical one that usually begins growing semi-elliptically. It has been reported by many authors that this crack is usually detected in welded joints subject to the residual stresses caused by welding. For example, Bergman [1] conducted a three-dimensional finite element evaluation of stress intensity factors for circumferential interior and exterior surface cracks front by the built-in J integral solver of ABAQUS software. Xiang-Ming et al. [2] determined the stress intensity factors for internal circumferential semi-elliptical cracks in cylinders, experimentally. Nabavi and Kamyab [3] computed the steady-state thermal stress intensity factors for an axisymmetric circumferential crack using the weight function method. The present paper investigates the steady-state thermal stress intensity factors at the deepest point of an internal circumferential semi-elliptical crack in a cylinder using the fundamentals of the weight function method. The cylinder has a radii ratio of $R_o/R_i=1.1$ and is subjected to thermo-mechanical stress distribution.

2- Methodology

Weight function is a general function for a cracked body that is independent of applied stress. The unique feature of the weight function method is that once the weight function has been determined, it can be used to compute the stress intensity factor of the crack under any stress distribution. Shen and

Glinka [4] provided the general form of the weight function for the deepest point of surface cracks containing four terms, as follows:

$$m(r,a) = \sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{R_i + a - r}} + 3\sqrt{\frac{2}{\pi}}\frac{1}{a}\sqrt{R_i + a - r} + M_1\sqrt{\frac{2}{\pi a}} + M_2\sqrt{\frac{2}{\pi a^3}}(R_i + a - r) \quad (1)$$

where the crack tip is at $r=R_i+a$. In order to define the weight function in the above equation, M_1 and M_2 have to be determined. Using Eq. (1), stress intensity factors for the deepest point of a circumferential semi-elliptical crack can be stated as follows:

$$K = \int_{R_i}^{R_i+a} \sigma(r)\, m(r,a)\, dr \quad (2)$$

where $\sigma(r)$ is the axial stress field of the prospective crack face in an uncracked structure and $m(r,a)$ is given in Eq. (1). Hence, the unknown parameters of the weight function can be determined using two reference loads as constant and linear pressure on crack face. Universality of the presented method is proved using the geometry correction factors Y_0 and Y_1 approximated by the closed form expressions as follows:

$$Y_j = \sum_{m=0}^{2}\sum_{n=0}^{3} A_{jmn}\,(a/t)^m\,(a/c)^n \qquad , \quad j = 0,\, 1 \quad (3)$$

where the parameters A_{jmn} are fitted to numerical data available in [5] and $j=1,2$ represents the uniform and linear load, respectively. So, using the above equations the unknown parameters of the weight function can be derived by the following equations:

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$$M_1 = -\sqrt{2/Q}\ \pi\ Y_0 + 3\sqrt{2/Q}\ \pi\ Y_1 - 24/5 \qquad (4\text{-}a)$$

$$M_2 = 3\sqrt{2/Q}\ \pi\ Y_0 - 6\sqrt{2/Q}\ \pi\ Y_1 + 8/5 \qquad (4\text{-}b)$$

where Q is the shape function. Consequently, using the derived weight function, thermal stress intensity factors of the crack can be calculated. To do so, substitution of the thermo-mechanical stress distribution in Eq. (2) as the longitudinal normal stress field applied to the crack face results in the closed form steady state thermal stress intensity factors at the deepest point of an internal circumferential semi-elliptical crack as a function of crack geometry, and loading conditions.

3- Discussion and Results

To investigate the behavior of the crack under thermo-mechanical loading the cylinder is subjected to internal and external pressures; $P_i = 10$ MPa , $P_o = 0.1$ MPa and the inner surface of the cylinder is subjected to convection cooling $\theta_s = -100$ °C. The Biot number ($Bi = R_i h/k$) is used to define the forced convection. Fig. 1 to 3 show the variation of dimensionless stress intensity factors at the deepest point of the cracked cylinder subjected to thermo-mechanical loading as a function of the Biot number. The results are compared with API standard code [6] for some specific conditions of thermal stress. Good agreement is achieved over the whole range of relative depths and aspect ratios. The results show that increasing the Biot number as a parameter to represent the forced convection heat transfer causes the stress intensity factor to increase. Furthermore, the maximum stress intensity factor for all crack geometries occurs when the Biot number is equal to infinity. This proves the fact that the conduction heat transfer mechanism provides a more conservative solution than the convective form. It is worth noting that increasing the relative depth of the circumferential semi-elliptical crack with a low Biot number gets the stress intensity factor to increase. This trend is completely reversed when the Biot number is high. Basically, in the case of cracks with low
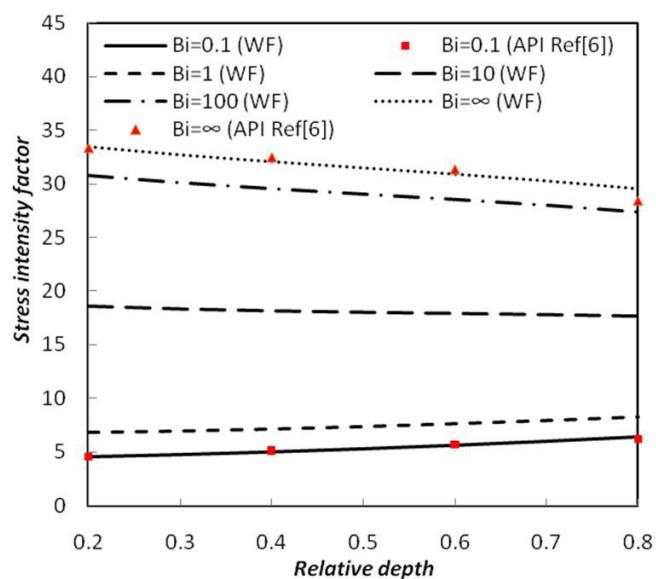

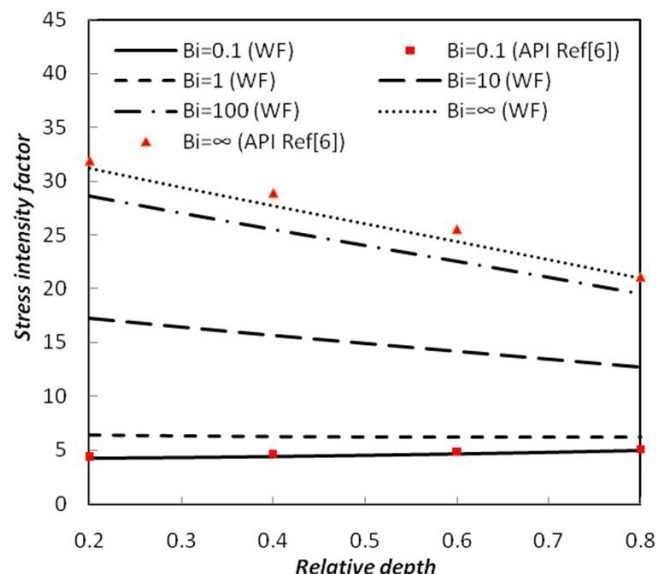**Figure 2. Variation of normalized thermal stress-intensity factor for *a/c*=0.5**
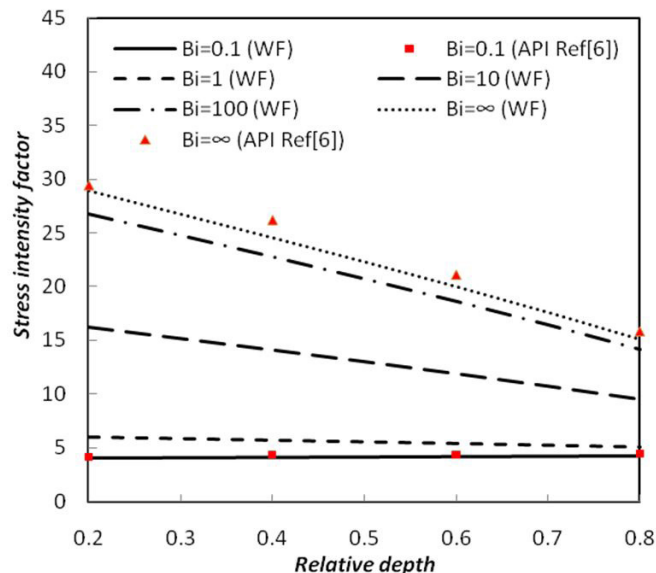

Figure 3. Variation of normalized thermal stress-intensity factor for *a/c*=1.0

aspect ratios the more the heat transfer inclined towards forced convection, increasing the relative depth makes the stress intensity factor decrease proving that shallower cracks are more critical than deeper ones. Finally, stress intensity factors decrease with increasing aspect ratio for all thermal loads situation. This means that semi-elliptical cracks are more critical than semi-circular ones.

4- Conclusions

This paper studied the stress intensity factors of circumferential internal semi-elliptical cracks in a cylinder subjected to thermo-mechanical boundary conditions using the weight function method. The results illustrated that the forced convection heat transfer solution gives lower stress intensity factors than the conduction one (applying temperature to the inner surface of the cylinder). Accordingly, the conduction

Figure 1. Variation of normalized thermal stress-intensity factor for *a/c*=0.25

solution is more conservative. Furthermore, it is shown that deeper cracks do not always have the maximum stress intensity factors; thus, for high aspect ratio cracks, as the heat transfer mechanism approaches forced convection, shallower cracks become more critical.

References

[1] Bergman, M., 1995. "Stress intensity factors for circumferential surface cracks in pipes". Fatigue and Fracture for Engineering Materials and Structures, 18(10), pp. 1155-1172.

[2] Xian-Ming K., Si-Tao Z., and Zhen-Yuan C., 1989. "Studies on stress intensity factor of surface cracks in a cylinder under remote tension loads", Engineering Fracture Mechanics. 33(1), pp. 105-111.

[3] Nabavi S.M., and Kamyab M., 2012. "Determination of transient thermal stress intensity factors for circumferential cracks in cylinders". Proc. 20th ISME mechanical engineering conference, Shiraz University, Shiraz, Iran.

[4] Glinka G., and Shen G., 1991. "Universal features of weight functions for cracks in mode I", Engineering Fracture Mechanics. 40, pp. 1135-1146.

[5] Zareei A., 2015. "Determination of weight function and stress intensity factors for circumferential semi-elliptical cracks in cylinders", MS Thesis, Faculty of Aerospace Engineering, Malek Ashtar University of Technology, Tehran, Iran.

[6] API 579-1/ASME FFS-1, 2007. Fitness-for-Service. 2nd ed.; American Petroleum Institute, Washington DC.