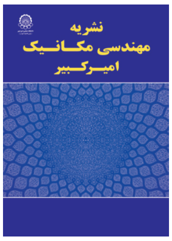

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Free Vibration Analysis of a Delaminated Beam with Stochastic Parameters

A.A. Alizadeh[1], R.-A. Jafari-Talookolaei[2*]

[1] Department of Mechanical Engineering, Isfahan University of Technology, Isfahan, Iran
[2] Department of Mechanical Engineering, Babol Noshirvani University of Technology, Babol, Iran

ABSTRACT: In this article, the random vibration analysis of a delaminated beam is performed for the first time by considering the thicknesswise location of the delamination and the Young's modulus as the stochastic parameters. First, the delaminated beam is divided into four intact sub-beams. Then by introducing a beam element and based on the beam's classical theory, the kinetic and potential energies of each sub-beam are derived. The considered higher order element has three nodes, one at each end and one at the midpoint and each node has two degrees of freedom, namely, deflection and slope. Using the mentioned energies, the stiffness and mass matrixes of the element are obtained. Next, by assembling the above stated matrices and considering the continuity conditions for adjoining elements at the delamination boundaries, the total stiffness and mass matrices are obtained. In employing the continuity conditions, the deflection and slope of these elements are taken to be equal. At the end, by applying the boundary conditions the governing differential equations of motion are obtained in matrix form. Then by modeling the stochastic parameters as random fields, the governing deterministic differential equation of the system is transformed into a stochastic differential equation. The continuous random fields are discretized by mid-point and local average discretization methods. Finally using the Monte Carlo simulation method in each iteration loop, each stochastic differential equation is transformed into a deterministic differential equation. For free vibration analysis, the eigenvalue problem is solved to investigate the frequencies and mode shapes of the system. Consequently, having the eigenpairs of the system, the statistical properties of free vibration characteristics of the beam such as expected values, standard deviations and probability density functions are obtained and the effect of different parameters of the beam and delamination are studied. Also in order to verify the obtained equations and the written computer programs, the deterministic frequencies of the beam are compared with other results and very good agreement is observed.

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1- Introduction

One of the most important damages that could affect vibrational properties of a beam is delamination. Until now, many studies have been done to determine the vibrational behavior of a delaminated beam with deterministic parameters for delamination [1-3], while, in many cases, it is impossible to specify the delamination properties precisely. Thus, random vibration analysis is necessary for a damaged beam with stochastic delamination parameters.

In this article, the free vibration analysis of a delaminated beam is investigated for the first time with probabilistic approach due to the uncertainty in the mechanical properties of the beam (Young's modulus) and thicknesswise location of the delamination. Also, each of the stochastic parameters is modeled as a random field and two different discretization methods are used to discretize them. Then, the Monte Carlo simulation method (MCSM) is used in conjunction with the finite element to obtain the statistical properties of the delaminated beam such as mean value, standard deviation and so on. On the other hand, vibration analysis of the delaminated beam with different boundary conditions is performed based on the free and constrained mode models.

2- Deterministic Formulation of a Delaminated Beam

In this article, higher order finite element is used to analyze the system. The displacement field of element could be obtained by Hermitian interpolation functions (Λ):

$$w = \sum_{i=1}^{3}\left(\Lambda_{2i-1}(\eta)w_i + \Lambda_{2i}(\eta)w'_i\right) = [\Lambda]\{\delta\} \qquad (1)$$

Corresponding author, E-mail: ra.jafari@nit.ac.ir

where w is vertical displacement and w' is the slope. Also, $\eta=x/L_e$ is the element local coordinate and $\{\delta\}$ is the degree of freedom vector. The mass and stiffness matrices could be obtained as below by using kinetic and potential energies.

$$\left[M^e\right] = \int_0^1 m_e [\Lambda]^T [\Lambda] L_e\, b\, d\eta \qquad (2)$$

$$\left[K^e\right] = \int_0^1 (EI)^e [\Lambda]_{,xx}^{\ T} [\Lambda]_{,xx} L_e\, b\, d\eta \qquad (3)$$

where m_e is the mass per unit area and $(EI)^e$ is the bending stiffness. Then, the equations of motion of the vibrating system can be obtained by assembling the mass and stiffness matrices and considering the continuity conditions at the delamination tips. By assuming a harmonic motion, we have:

$$\left([K] - \omega^2 [M]\right)\{a_0\} = \{0\} \qquad (4)$$

3- Probabilistic Analysis of the Delaminated Beam

The governing equations of motion are transformed into the stochastic differential equation by assuming the Young's modulus and the thicknesswise location of the delamination as the stochastic parameters. On the other hand, these stochastic parameters are modeled as the random fields. These random fields are discretized by the mid-point and local average methods in order to use them in finite element modeling [4]. After discretizing a random field, the governing stochastic differential equation is solved by MCSM [5]. The main advantage of this method is to obtain accurate solutions using large numbers of simulations for all the problems for which deterministic solutions exist. In this method, each random parameter is converted to a random vector using the random field discretization methods and with respect to the correlation function of that random field. Then, every realized random vector corresponding to each random field is substituted in different elements. Afterwards, using FEM, a deterministic ordinary differential equation is obtained and solved. This process is repeated for many realizations. Finally, probabilistic responses of the problem under study, such as mean values, standard deviations, probability density functions etc. could be obtained from the achieved responses in different realizations.

4- Numerical Results

At first, the deterministic analysis of the system is investigated to validate the developed model and it is shown that the results are the same as those of Mujumdar and Suryanarayan [2]. Table 1 shows the mean values of the first and second mode frequencies for different delamination length (L_2) and also different mean value of thicknesswise location ($\bar{h}_2$). As it is clear, the difference between the free (F) and constrained (C) mode frequencies increases by decreasing $\bar{h}_2$ and increasing . Also, there is a little difference between the results obtained by mid-point and local average methods.

Fig. 1 and Fig. 2 show the mean and standard deviation values of the first mode shape of the beam with delamination parameters $L_2=0.8L$, $\bar{h}_2=0.2h$ respectively. According to Fig. 1, the delamination opening phenomenon is observed due to the closeness of the delamination to the free surface and also its long length.

Table 1. Mean values of the first and second mode frequencies for different delamination parameters

Local average $L_2=0.8L$ $\bar{h}_2=0.2h$		Mid-point $L_2=0.2L$ $\bar{h}_2=0.5h$		Local average $L_2=0.2L$ $\bar{h}_2=0.5h$		
C	F	C	F	C	F	
0.404	0.402	0.45	0.449	0.449	0.448	1st mode
2.486	2.032	2.332	2.325	2.33	2.321	2nd mode

On the other hand, as it is obvious from Fig. 2, this opening has significant effect on the standard deviation of sub-beams 2 and 3.

5- Conclusions

The following conclusions can be drawn from the results presented in this study:

- The difference between the free and constrained mode frequencies is increased by decreasing its thicknesswise location and by increasing its length.
- Mean values of the first mode frequencies have relatively significant difference with the deterministic results for the long length and close to free surface delamination.
- Standard deviations of the frequencies for mid-point method are slightly greater than those predicted by the local average method.
- Standard deviations of the frequencies are equal for free and constrained mode models.
- The opening phenomenon in the delamination region can be occurred by increasing the delamination length and decreasing its height.
- The opening phenomenon has a significant effect on the standard deviation values.
- Mean value of the first mode frequencies decreases and its standard deviation increases by increasing the standard deviations of input stochastic parameters.

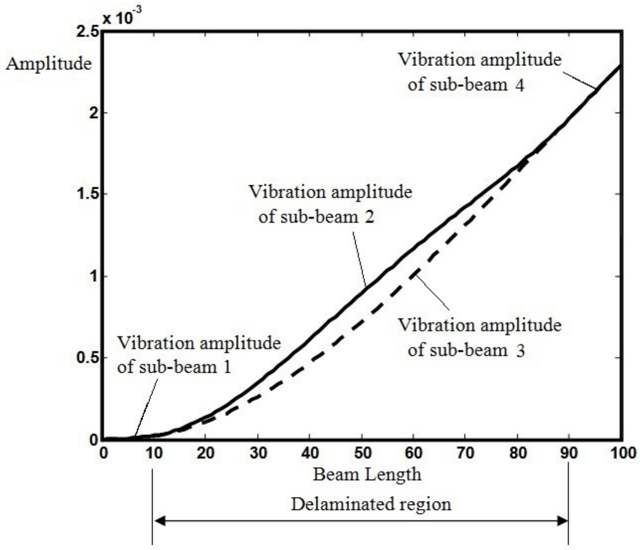

Figure 1. Mean values of the first mode shape

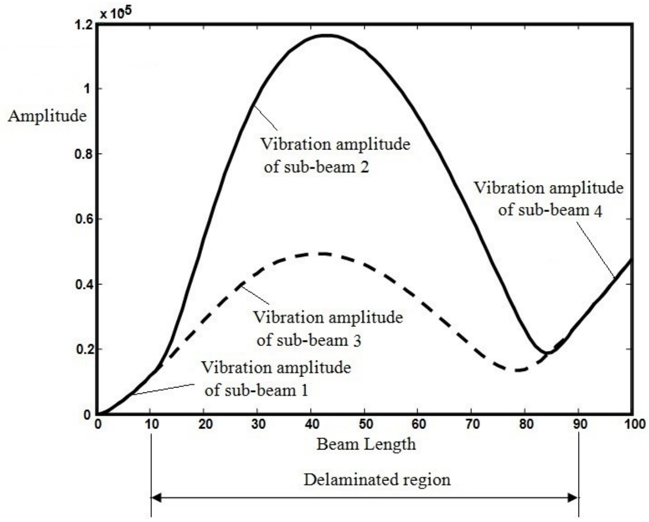

Figure 2. Standard deviation of the first mode shape

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