

# Flutter Instability of Aircraft Swept Wings by Using Fully Intrinsic Equations 

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#### Abstract

In this paper, the dynamic instability of swept wings by using the geometrically exact fully intrinsic beam equations is investigated. Due to the lack of existence of sweep angle effects in the aeroelastic formulation of these equations, this study is aimed to add the effect of sweep angle to the aforementioned formulation and this is one of the aspects of innovation in this paper. The fully intrinsic equations involve only moments, forces, velocity and angular velocity, and in these equations, the displacements and rotations will not appear explicitly. For this reason, the important advantages of these equations are complete modeling without any simplifying assumptions in large deformations, low-order nonlinearities and thus less complexity. In order to determine the stability of the wing, first the resultant non-linear partial differential equations are discretized by using the central finite difference method, and then linearized about the static equilibrium. Afterward, using the eigenvalue analysis of linearized equations, the stability of the system versus different parameters is evaluated. The obtained results are compared with those available in the literature, and good agreement is observed. Finally, it is observed that by using the fully intrinsic equations, the instability of the swept wings can be determined accurately.


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## 1- Introduction

Aircraft wing structures are conventionally modeled by beam models. There are different beam models which described by various beam equations and theories. Assumptions used in these equations often resulting in limitations, complexities and reduction of accuracy. Geometrically exact fully intrinsic beam equations are one of the most accurate beam equations. These equations comprise a set of geometrically exact, nonlinear, first-order partial differential equations which are suitable for large deformations.
In the present study, the dynamic instability of swept wings by using the geometrically exact fully intrinsic beam equations is investigated. Due to the lack of existence of sweep angle effects in the aeroelastic formulation of these equations, this study is aimed to add the effect of sweep angle to this formulation and this is one of the aspects of innovation in this paper.

## 2- Nonlinear, Fully Intrinsic Beam Equations

The geometrically exact, intrinsic governing equations for the dynamics of a general, non uniform, twisted, curved, anisotropic beam are [1]

$$
\begin{align*}
& F_{B}^{\prime}+\tilde{K}_{B} F_{B}+f_{B}=\dot{P}_{B}+\tilde{\Omega}_{B} P_{B} \\
& M_{B}^{\prime}+\tilde{K}_{B} M_{B}+\left(\tilde{e}_{1}+\tilde{\gamma}\right) \Omega_{B}+m_{B} \\
& \quad=\dot{H}_{B}+\tilde{\Omega}_{B} H_{B}+\tilde{V}_{B} P_{B}  \tag{1}\\
& V_{B}^{\prime}+\tilde{K}_{B} V_{B}+\left(\tilde{e}_{1}+\tilde{\gamma}\right) \Omega_{B}=\dot{\gamma} \\
& \Omega_{B}^{\prime}+\tilde{K}_{B} \Omega_{B}=\dot{k}
\end{align*}
$$

[^0]where ( )' denotes the derivative with respect to the undeformed beam reference line and () denotes the absolute time derivative, $F_{B}$ and $M_{B}$ are the internal force and moment measures, $P_{B}$ and $H_{B}$ are the sectional linear and angular momenta, $V_{B}$ and $\Omega_{B}$ are the velocity and angular velocity measures, and $\gamma$ and $\kappa$ are the force and moment strain measures, $K_{B}=k_{b}+\kappa$ is the curvature vector and $k_{b}$ is the initial twist and curvature of the beam. $f_{B}$ and $m_{B}$ include all the external forces and moments such as gravity, aerodynamic forces and moments, and control forces and moments.
The generalized forces $\left(F_{B}, M_{B}\right)$ and the generalized strains $(\gamma, \kappa)$ are related to each other via the beam cross-section stiffness matrix. The beam cross-section inertia matrix leads to the relation between the generalized momenta ( $\mathrm{P}, \mathrm{H}$ ) and the generalized velocities $(\mathrm{V}, \Omega)$.
\[

\left\{$$
\begin{array}{l}
\gamma  \tag{2}\\
k
\end{array}
$$\right\}=\left[$$
\begin{array}{ll}
R & S \\
S^{T} & T
\end{array}
$$\right]\left\{$$
\begin{array}{l}
F_{B} \\
M_{B}
\end{array}
$$\right\}
\]

where $R, S$, and $T$ are the cross-sectional flexibility matrix.
$\left\{\begin{array}{l}P_{B} \\ H_{B}\end{array}\right\}=\left[\begin{array}{cc}\mu \Delta & -\mu \tilde{\xi} \\ \mu \tilde{\xi} & I\end{array}\right]\left\{\begin{array}{l}V_{B} \\ \Omega_{B}\end{array}\right\}$
where $\mu$ is the mass per unit length, $\Delta$ is the identity matrix, $\xi$ is the cross-sectional mass centroid offset, and $I$ is the inertia matrix per unit length. [2]

## 3- Peters Aerodynamic Model

The aerodynamic loads on the wing in an incompressible flow regime are determined by using the Peters unsteady aerodynamic model. The aerodynamic force and moment can
be written as [3]
$f_{a}^{n}=\rho b^{n}\left\{\begin{array}{c}f_{a 1}^{n} \\ f_{a 2}^{n} \\ f_{a 3}^{n}\end{array}\right\}$
$m_{a}^{n}=2 \rho b^{n^{2}}\left\{\begin{array}{c}m_{a 1}^{n} \\ m_{a 2}^{n} \\ m_{a 3}^{n}\end{array}\right\}$
where
$f_{a 1}^{n}=0$
$f_{a 2}^{n}=-C_{l_{0}}^{n} V_{T}^{n} V_{a_{2}}^{n}+C_{l_{\alpha}}^{n}\left(V_{a_{3}}^{n}+\lambda_{0}^{n}\right)^{2}-C_{d_{0}}^{n} V_{T}^{n} V_{a_{2}}^{n}$
$f_{a 3}^{n}=C_{l_{0}}^{n} V_{T}^{n} V_{a_{3}}^{n}-C_{l_{\alpha}}^{n} \dot{V}_{a_{3}}^{n} b / 2-$

$$
C_{l_{\alpha}}^{n} V_{a_{2}}^{n}\left(V_{a_{3}}^{n}+\lambda_{0}^{n}-\Omega_{a_{1}}^{n} b^{n} / 2\right)-C_{d_{0}}^{n} V_{T}^{n} V_{a_{3}}^{n}
$$

$m_{a 1}^{n}=C_{m_{0}}^{n} V_{T}^{n^{2}}-C_{m_{\alpha}}^{n} V_{T}^{n} V_{a_{3}}^{n}-b^{n} C_{l_{\alpha}}^{n} / 8 V_{a_{2}}^{n} \Omega_{a_{1}}^{n}-$

$$
b^{n^{2}} C_{l_{\alpha}}^{n} \dot{\Omega}_{a_{1}}^{n} / 32+b^{n} C_{l_{\alpha}}^{n} \dot{V}_{a_{3}}^{n}
$$

$m_{a 2}^{n}=0$
$m_{a 3}^{n}=0$
$\lambda_{0}^{n}=\frac{1}{2}\left\{b_{\text {inflow }}\right\}^{T}\left\{\lambda^{n}\right\}$
$\left[A_{\text {inflow }}\right]\left\{\dot{\lambda}^{n}\right\}+\left(\frac{V_{T}^{n}}{b^{n}}\right)\left\{\lambda^{n}\right\}=$

$$
\left(\dot{V_{a_{3}}^{n}}+\frac{b^{n}}{2} \dot{\Omega}_{a_{1}}^{n}\right)\left\{c_{\text {inf low }}\right\}
$$

$\overline{V_{a}^{n}}=C_{a}^{n T} \overline{V^{n}}-\tilde{y}_{m c}^{n} C_{a}^{n T} \bar{\Omega}^{n}$
$\bar{\Omega}_{a}{ }^{n}=C_{a}^{n T} \bar{\Omega}^{n}$
and $y_{m c}{ }^{n}$ is a row matrix containing the measures of the position vector from the beam reference axis to the mid-chord. Also, $\lambda^{n}$ is a column matrix of inflow states for the $n$th element, and $\left[A_{\text {inf low }}\right],\left[b_{\text {inf low }}\right],\left[c_{\text {inflow }}\right]$ are constant matrices derived in [4]. $c_{a}$ is the direction cosine matrix of deformed frame with respect to aerodynamic frame [5].

## 4- Solution Procedure

In the present study finite difference discretizing in time and space are used for solving the governing equations.
These discretized equations linearized about the static equilibrium and by using the eigenvalue analysis of linearized equations, the stability of the system versus different parameters are evaluated.

## 5- Results and Discussion

To make a comparison between the present results and the results reported in the literature, two different wings are considered and good agreement is observed. Fig. 1 shows that there is a good agreement between the present results and reference [6].
Fig. 2 shows the variations of flutter speed with sweep angle and stiffness ratio $(\psi)$.


Figure 1. Variations of flutter speed with sweep angle


Figure 2. Variations of flutter speed with stiffness ratio and sweep angle

## 6- Conclusions

Because of the important effects of sweep angle on the flutter speed of a wing, finding the effect of different parameters simultaneously with changing the sweep angle on flutter speed is considered in this article.
Results show that the flutter speed strictly depends on the sweep angle, altitude, mass ratio, wing stiffness, radius of gyration, center of gravity location and aspect ratio of the wing.

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