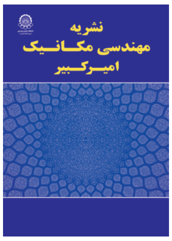

Amirkabir Journal of Mechanical Engineering

Amirkabir Journal of Mechanical Engineering, 49(2) (2017) 113-116
DOI: 10.22060/mej.2016.742

Analytical Investigation of Nonlinear Free Vibration of Magneto-Electro-Elastic Rectangular Thin Plate Resting on a Nonlinear Elastic Foundation

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ABSTRACT: In this paper, nonlinear free vibration of magneto-electro-elastic rectangular thin plate is investigated. The plate is supported by a nonlinear foundation and simply-supported boundary condition is assumed along all edges. The plate is considered in two forms; uniformly distributed one-layered plate and the functionally graded one. The plate is subjected to electric and magnetic potentials between top and bottom surfaces. Equations of motion of this smart plate are obtained by using classical plate theory along with the Gauss laws for electrostatics and magnetostatics. Then, the obtained equation of motion is analytically solved by using multiple time scales method. Effects of several parameters like plate's dimension, foundation parameter and electric and magnetic potentials on the nonlinear response of the plate are studied.

Review History:

Received: 29 June 2015
Revised: 31 October 2015
Accepted: 7 February 2015
Available Online: 8 November 2016

Keywords:

Functionally graded smart plate
magneto-electric coupling
classical plate theory
multiple time scales method

1- Introduction

In recent years, magneto-electro-elastic materials have been the topic of many researches due to their ability to transform electrical, magnetic, and mechanical energy forms to each other. Pan [1] studied response of a laminated magneto-electro-elastic plate analytically for the first time. Li and Zhang [2] used Mindlin theory to determine natural frequencies of a magneto-electro-elastic plate resting on an elastic foundation. Xue et al. [3] analyzed large deflection of a magneto-electro-elastic thin plate based on the classical plate theory.

In this paper, effects of several parameters on the nonlinear free vibration of a functionally graded magneto-electro-elastic plate are investigated based on the classical plate theory in conjunction with single-mode Galerkin method and multiple time scales method.

2- Modelling the Problem

Constitutive equations of a magneto-electro-elastic material are expressed by Xue et al. [3]:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}=\begin{bmatrix}C_{11} & C_{12} & 0\\ C_{12} & C_{22} & 0\\ 0 & 0 & C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}+\begin{bmatrix}0 & 0 & e_{31}\\ 0 & 0 & e_{31}\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}0\\ 0\\ \phi_{,z}\end{Bmatrix}+\begin{bmatrix}0 & 0 & q_{31}\\ 0 & 0 & q_{31}\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}0\\ 0\\ \psi_{,z}\end{Bmatrix} \quad (1)$$

$$\begin{Bmatrix}D_z\\ B_z\end{Bmatrix}=\begin{bmatrix}e_{31} & e_{31}\\ q_{31} & q_{31}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\end{Bmatrix}-\begin{bmatrix}\eta_{33} & d_{33}\\ d_{33} & \mu_{33}\end{bmatrix}\begin{Bmatrix}\phi_{,z}\\ \psi_{,z}\end{Bmatrix} \quad (2)$$

where C_{ij}, e_{31}, q_{31}, η_{33}, d_{33}, and μ_{33} are stiffness coefficient, piezoelectric, piezomagnetic, dielectric, magneto-electric, and magnetic permeability constants, respectively. σ_i, ϕ, D_z, ψ, and B_z denote stress, electric potential, electric displacement along z-axis, magnetic potential, and magnetic flux density along z-axis, respectively.

The plate is $CoFe_2O_4$-rich at $z = -h/2$ and $BaTiO_3$-rich at at $z = +h/2$, and the material properties are changed along z-axis. Volume fraction of the piezoelectric phase (i.e., $BaTiO_3$) is determined by:

$$V_B=\left[(2z+h)/(2h)\right]^N \quad (3)$$

where N is a non-negative number and B denotes the piezoelectric phase. Then, material properties of the plate can be obtained by the following equation:

$$U=(U_B-U_F)V_B+U_F \quad (4)$$

where U denotes C_{ij}, e_{31}, q_{31}, or ρ_0, and F represents the piezomagnetic phase. to obtain closed-form expression for the nonlinear frequency of the plate, η_{33} and μ_{33} are assumed to be independent of z and determined by:

$$\eta_{33}=\left(\eta_{33_B}+\eta_{33F}\right)/2,\ \mu_{33}=\left(\mu_{33_B}+\mu_{33F}\right)/2 \quad (5)$$

Using Gauss's laws for electrostatics and magnetostatics, i.e.,

$$D_{z,z}=0, \qquad B_{z,z}=0 \quad (6)$$

$\phi_{,z}$ and $\psi_{,z}$ in Eq. 1 are obtained and consequently the resultants can be determined in terms of displacements:

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$$\begin{aligned}
N_{xx} &= \hat{N}_1 w_{,x}^2 + \hat{N}_2 w_{,y}^2 + \hat{N}_3 u_{,x} + \hat{N}_4 v_{,y} + \\
&\quad \hat{N}_5 w_{,xx} + \hat{N}_6 w_{,yy} + f_1(V\,\Omega), \\
N_{yy} &= \hat{N}_7 w_{,x}^2 + \hat{N}_8 w_{,y}^2 + \hat{N}_9 u_{,x} + \hat{N}_{10} v_{,y} + \\
&\quad \hat{N}_{11} w_{,xx} + \hat{N}_{12} w_{,yy} + f_1(V\,\Omega), \\
N_{xy} &= \hat{N}_{13} u_{,y} + \hat{N}_{14} v_{,x} + \hat{N}_{15} w_{,x} w_{,y} + \hat{N}_{16} w_{,xy} \\
M_{xx} &= \hat{M}_1 w_{,x}^2 + \hat{M}_2 w_{,y}^2 + \hat{M}_3 u_{,x} + \hat{M}_4 v_{,y} + \\
&\quad \hat{M}_5 w_{,xx} + \hat{M}_6 w_{,yy} + f_2(V\,\Omega), \\
M_{yy} &= \hat{M}_7 w_{,x}^2 + \hat{M}_8 w_{,y}^2 + \hat{M}_9 u_{,x} + \hat{M}_{10} v_{,y} + \\
&\quad \hat{M}_{11} w_{,xx} + \hat{M}_{12} w_{,yy} + f_2(V\,\Omega), \\
M_{xy} &= \hat{M}_{13} u_{,y} + \hat{M}_{14} v_{,x} + \hat{M}_{15} w_{,x} w_{,y} + \\
&\hat{M}_{16} w_{,xy}
\end{aligned} \tag{7}$$

where the coefficients are functions of plate's properties and are avoided here for brevity.

Equations of motion of a plate based on the classical plate theory are expressed by Reddy [4]:

$$N_{xx,x} + N_{xy,y} = 0 \tag{8}$$

$$N_{xy,x} + N_{yy,y} = 0 \tag{9}$$

$$\begin{aligned} &M_{xx,xx} + 2M_{xy,xy} + M_{yy,yy} + N_{xx} w_{,xx} + \\ &2N_{xy} w_{,xy} + N_{yy} w_{,yy} - k_{NL} w^3 = I_0 \ddot{w} \end{aligned} \tag{10}$$

Substituting Eq. 7 into Eqs. 8-10, using the following trial functions for displacements

$$u = hU(t)\sin(2\pi x/a)\sin(\pi y/b) \tag{11}$$

$$v = hV(t)\sin(\pi x/a)\sin(2\pi y/b) \tag{12}$$

$$w = hW(t)\sin(\pi x/a)\sin(\pi y/b) \tag{13}$$

and then by applying the Galerkin method on the resulted equations, one can obtain the following nonlinear differential equation:

$$\ddot{W} + \omega_0^2 W + \beta W^2 + \alpha W^3 = 0 \tag{14}$$

where ω_0 is the natural frequency of the plate.

Following the procedure presented by Nayfeh and Mook [5], Eq. 14 is solved and the nonlinear frequency is obtained:

$$\omega_{NL} = \omega_0 + \tfrac{1}{8} r_0^2 \left(\tfrac{3}{\omega_0} \alpha - \tfrac{10}{3\omega_0^3} \beta^2 \right) \varepsilon^2 \tag{15}$$

from which the nonlinear frequency to linear frequency ratio can be determined.

3- Results and Discussion

To validate the proposed solution, nonlinear frequency ratio of an isotropic square plate is obtained by the present method and the results are compared with the published ones. The results are shown in Table 1.

In Table 2 natural frequencies of functionally graded magneto-electro-elastic square plate are given. Figure 1 shows the backbone curve functionally graded magneto-electro-elastic plate.

Table 1. Nonlinear frequency ratio of an isotropic square plate.

Method	r_0		
	0.2	0.6	1.0
Shi et al. [6]	1.0195	1.1658	1.4163
Present	1.0190	1.1598	1.3995

Table 2. Natural frequencies (rad/s) of functionally graded magneto-electro-elastic plate ($a = b = 100h$).

Method		N			
		B	1	2	F
V_0 (V)	0	3.06113	3.62186	3.74978	4.19752
	10^3	3.06103	3.62175	3.74967	4.19852
Ω_0 (A)	0	3.06113	3.62186	3.74978	4.19752
	1	3.06113	3.62187	3.74979	4.19853

It is seen from Table 2 that positive electric potential decreases the natural frequency, whereas the positive magnetic potential increases it. Moreover, Figure 1 shows that by increasing the index of volume fraction (N), backbone curve bend away more from the vertical axis.

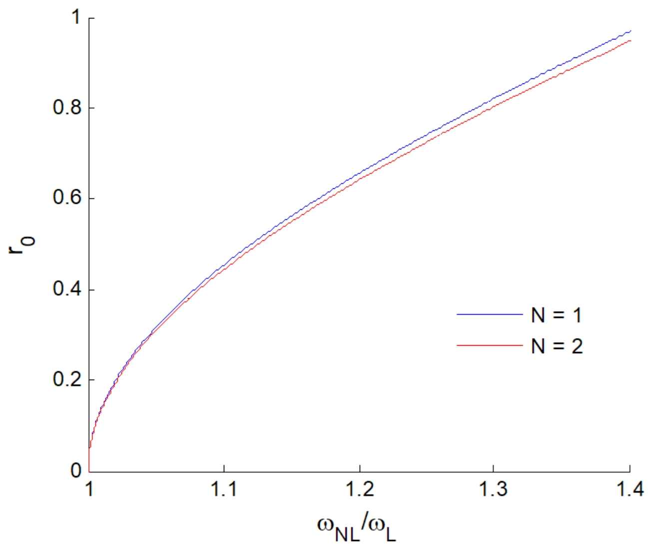

Figure 1. Backbone curve of functionally graded magneto-electro-elastic plate

4- Conclusions

In this paper, nonlinear free vibration of functionally graded magneto-electro-elastic plate is investigated based on the classical plate theory along with multiple time scales method. Several examples are given to validate the proposed solution and to investigate the effects of some parameters on the vibration response of this plate.

References

[1] [1] E. Pan, Exact solution for simply supported and multilayered magneto-electro-elastic plates, *TRANSACTIONS-AMERICAN SOCIETY OF MECHANICAL ENGINEERS JOURNAL OF APPLIED MECHANICS*, 68(4) (2001) 608-618.

[2] Y. Li, J. Zhang, Free vibration analysis of magnetoelectroelastic plate resting on a Pasternak foundation, *Smart materials and structures*, 23(2) (2013) 025002.

[3] C. Xue, E. Pan, S. Zhang, H. Chu, Large deflection of a rectangular magnetoelectroelastic thin plate, *Mechanics Research Communications*, 38(7) (2011) 518-523.

[4] J.N. Reddy, Mechanics of laminated composite plates and shells: theory and analysis, *CRC press*, 2004.

[5] A.H. Nayfeh, D.T. Mook, *Nonlinear oscillations*, John Wiley & Sons, 2008.

[6] Y. Shi, R.Y. Lee, C. Mei, Finite element method for nonlinear free vibrations of composite plates, *AIAA journal*, 35(1) (1997) 159-166.

Please cite this article using:

S. Razavi and A. Shooshtari, “Analytical Investigation of Nonlinear Free Vibration of Magneto-Electro-Elastic Rectangular Thin Plate Resting on a Nonlinear Elastic Foundation” *Amirkabir J. Mech. Eng.*, 49(2) (2017) 317-324.

DOI: 10.22060/mej.2016.742